PhD Defense

Symbolic Proofs of Computational Indistinguishability

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Thèse préparée au sein du LSV, ENS Paris-Saclay

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Introduction

Motivation

Security Protocols

Distributed programs which aim at providing some **security** properties.

Security Properties

The Problem

Attacks against security protocols can be very **damageable**, e.g. theft or privacy breach.

$\Rightarrow$ We need to check that protocols are secure.

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- Security protocols may be **short**: few lines of specification.

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The Context

- Security protocols may be **short**: few lines of specification.
- Security properties are **complex**.

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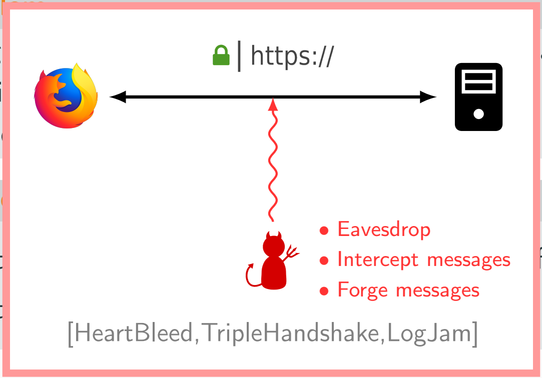
⇒ We nee

The Cont

- Securit fication.
- Securit



Can We Use Testing?

Principle

Run the protocol **multiple** times, on **random inputs**, to look for bugs.

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Problem

A protocol is not executed in a random environment:
an adversary can systematically trigger an unlikely corner case.

Formal Verification

Goal

Provide a **mathematical proof** that a **protocol** P is **secure**:

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$$\mathsf{P} \models \phi_{\mathsf{safe}}$$

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Provide a **mathematical proof** that a **protocol** P is **secure**:

$\forall$  $\in \mathcal{C} \qquad ($  $\mid\mid \mathrm{P}) \models \phi_{\mathsf{safe}}$

Question

What is the class of attackers $\mathcal{C}$?

Symbolic Attackers

Dolev-Yao Model

- Symbolic model, messages are (first-order) **terms**:

$$t = \{\langle \mathsf{A} , \mathsf{n_A} \rangle\}_{\mathsf{pk_B}}$$

- The adversary is explicitly granted some **capabilities**, e.g.:

$$\frac{a \qquad b}{\langle a , b \rangle} \qquad\qquad \frac{m \qquad \mathsf{pk}}{\{m\}_{\mathsf{pk}}}$$

$$\frac{\langle a , b \rangle}{a} \qquad\qquad \frac{\langle a , b \rangle}{b} \qquad\qquad \frac{\{m\}_{\mathsf{pk}} \qquad \mathsf{sk}}{m}$$

Symbolic Attackers

Advantages

- Adapted to proof automation: ProVerif, Tamarin, Deepsec...
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Problem

We prove only that there are no attacks **using the capabilities granted to the attacker**.

Computational Attackers

Computational Model

- More realistic model, messages are **bit-strings**.
- The attacker is any **Probabilistic Polynomial-time Turing Machine** (PPTM).
- The security property is expressed through a **game**.

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This model gives **strong security guarantees**.

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Problems

- Proofs are long, complicated and error-prone.
- Implicit hypotheses.
  Example: *An agent name cannot be confused with a pair*.
- Proof automation is hard (CryptoVerif).

The Bana-Comon Model

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$$\frac{\mathsf{len}(u) = \mathsf{len}(v)}{\{u\}_{\mathsf{pk}} \sim \{v\}_{\mathsf{pk}}} \ \mathsf{CPA}$$

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- We have to prove that the axioms **entail** the security property.

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Advantages

- This model gives **strong security guarantees**.
- **Formal model**, which may be amenable to **automated deduction** techniques.
- All hypotheses are **explicit** (in the axioms).

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Variants

- A **reachability** logic, studied in Scerri's thesis.
- A more recent **indistinguishability** logic.

The Bana-Comon Model

Problems at the Beginning of this Thesis

- Usefulness remained to be shown:
  - lack of case studies (only a toy example).
  - small set of axioms.
- No proof automation.

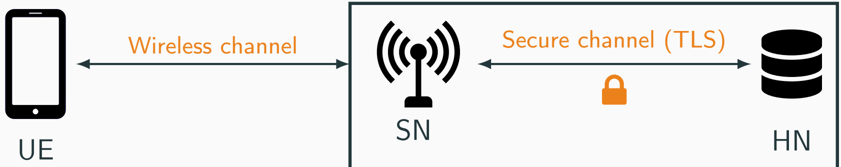
This Thesis

Contributions

- **Case study** of two RFID protocols, KCL and LAK.
- **Case study** of a complex protocol, AKA.
- **Decidability result** for a fixed set of axioms.

The AKA Protocol

Authentication and Key Agreement Protocol



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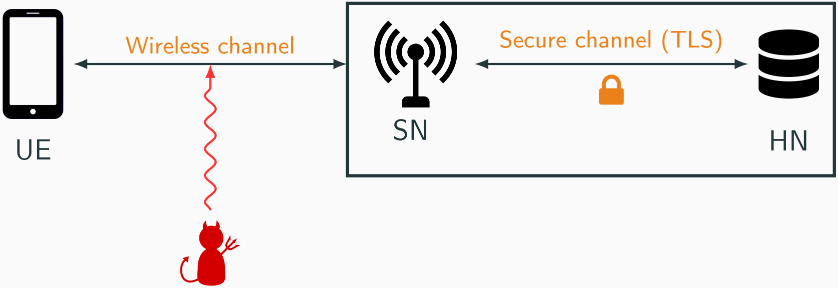

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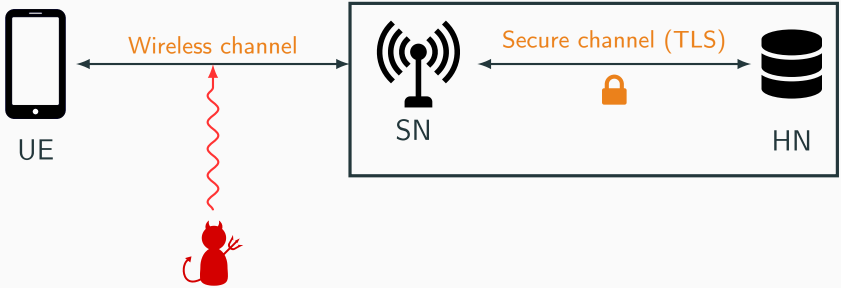
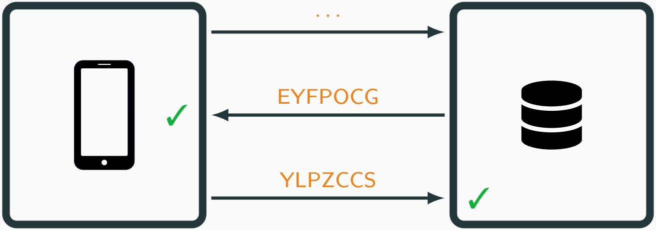


Security Properties

- **Mutual authentication** between the user and the service provider.
- **Untraceability** of the user against an outside observer.

Replay Protection



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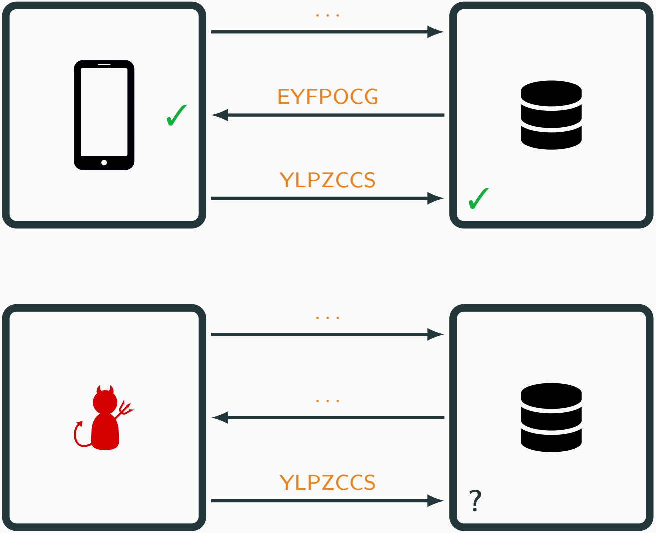

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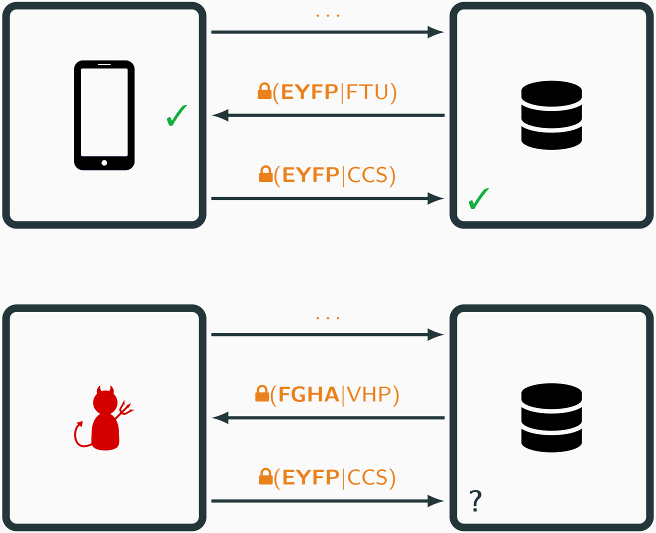

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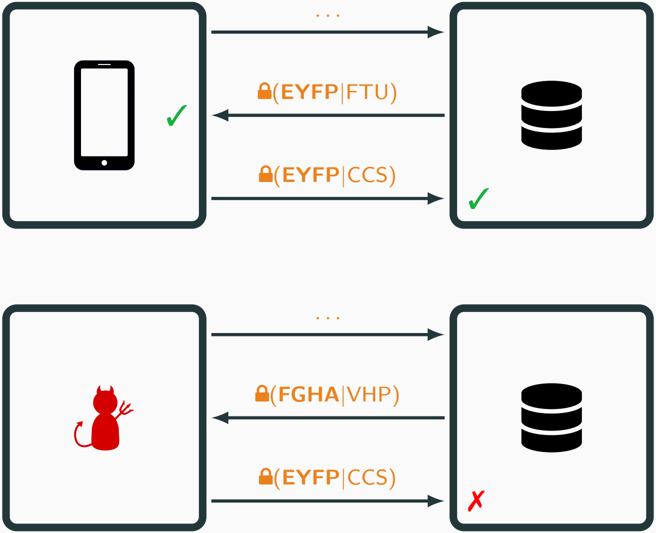

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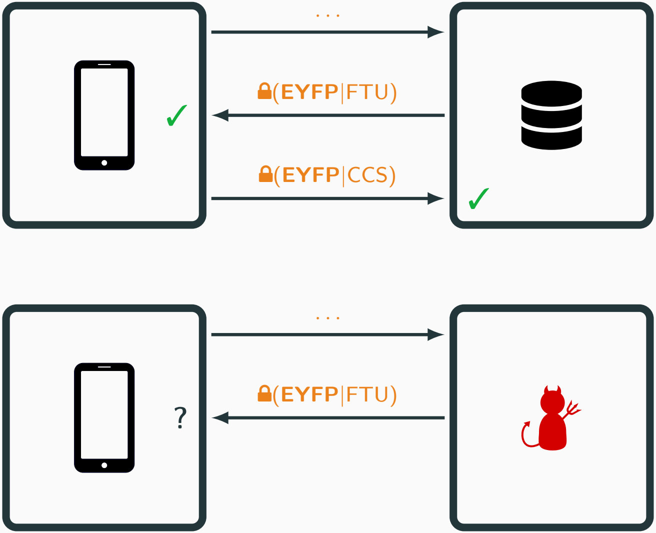

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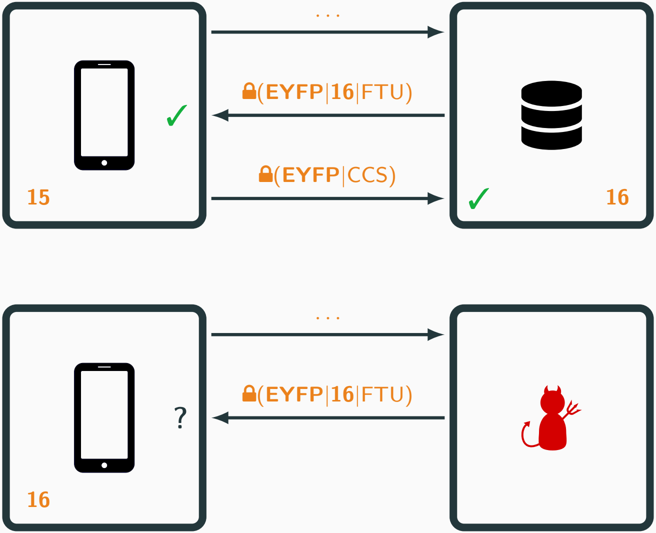

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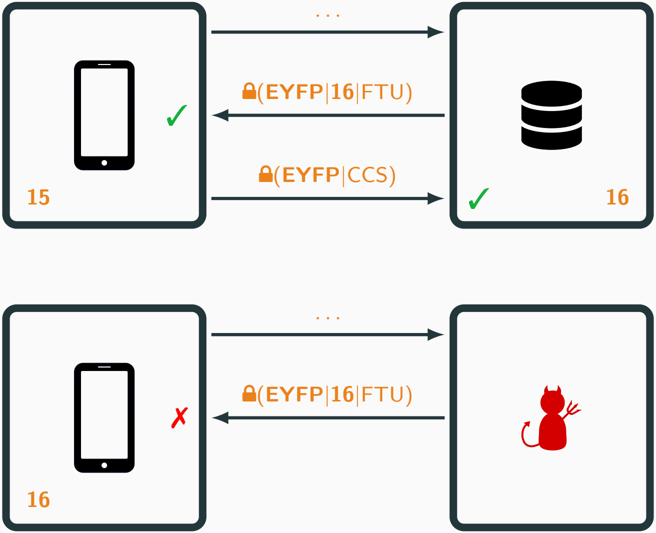
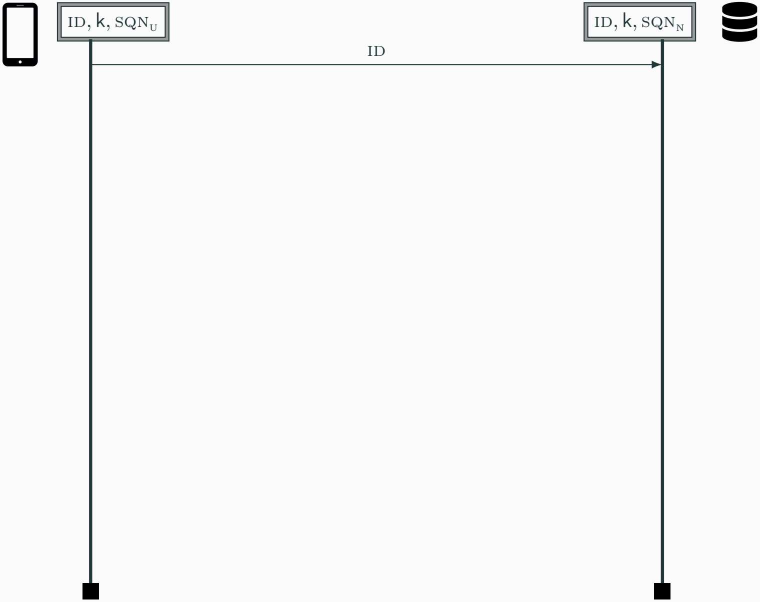
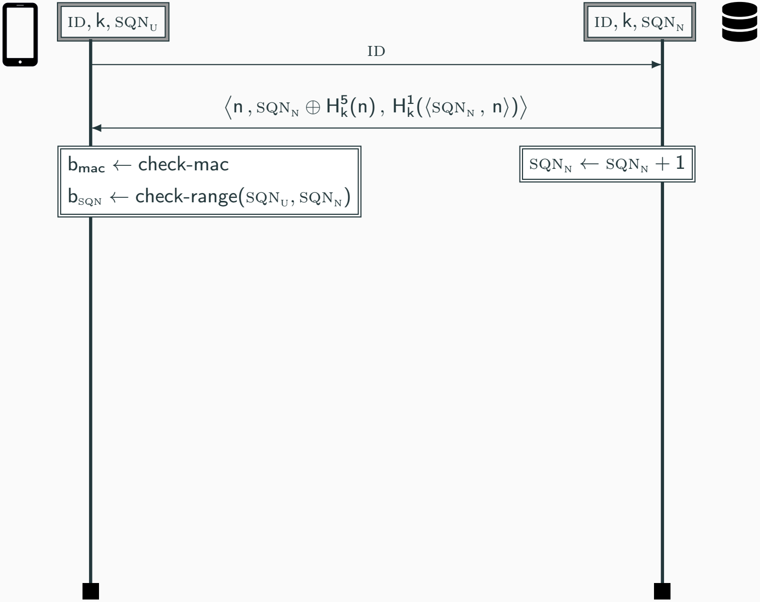




4G-AKA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ID, k, SQN_N
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b_SQN ← check-range(SQN_U, SQN_N)
SQN_N ← SQN_N + 1

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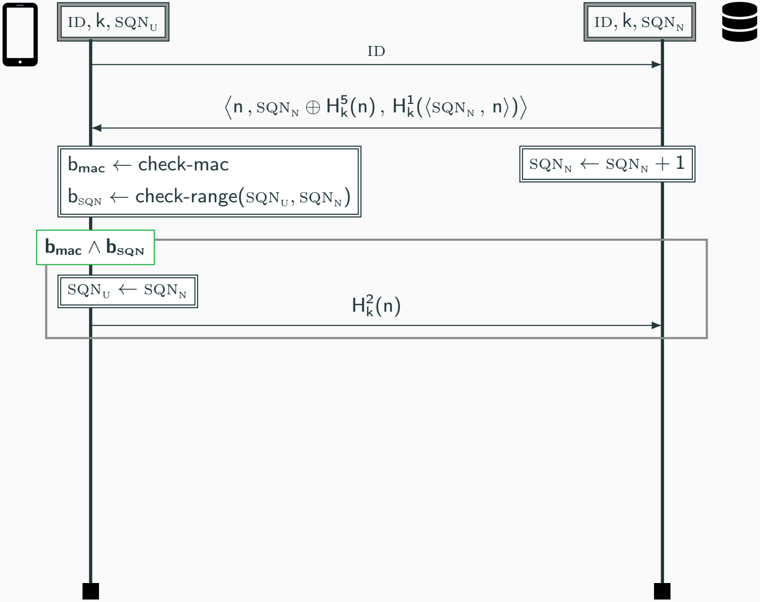
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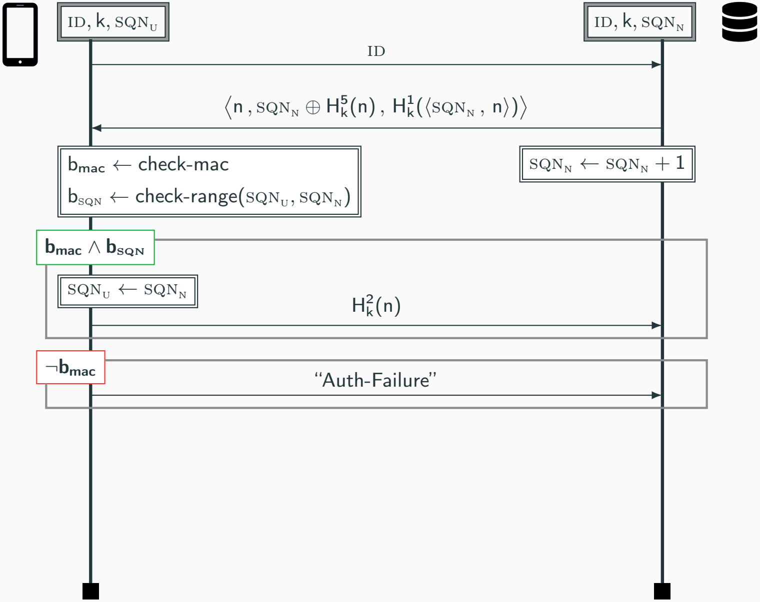
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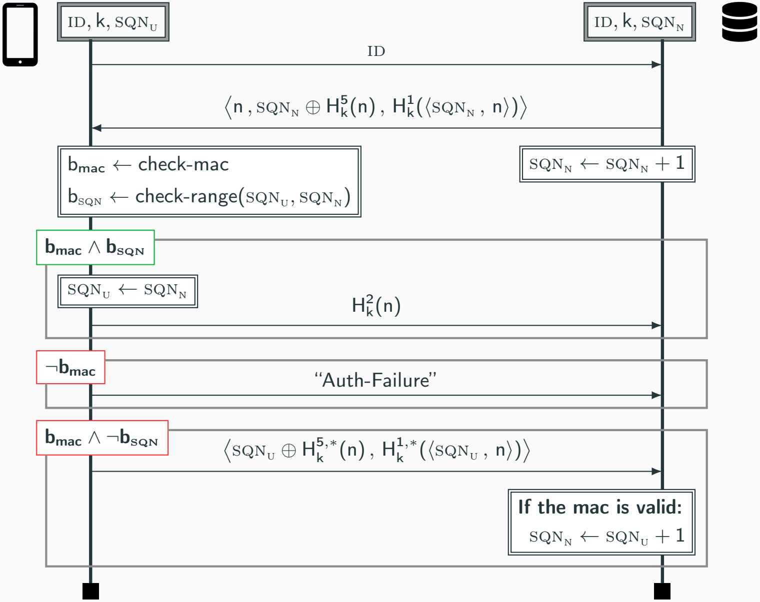

$\mathrm{ID}, \mathsf{k}, \mathrm{SQN}_U$
$\mathrm{ID}, \mathsf{k}, \mathrm{SQN}_N$
ID
$\langle \mathsf{n}, \mathrm{SQN}_N \oplus \mathsf{H}^5_\mathsf{k}(\mathsf{n}), \mathsf{H}^1_\mathsf{k}(\langle \mathrm{SQN}_N, \mathsf{n}\rangle)\rangle$
$\mathsf{b}_{\mathsf{mac}} \leftarrow$ check-mac
$\mathsf{b}_{\mathrm{SQN}} \leftarrow$ check-range$(\mathrm{SQN}_U, \mathrm{SQN}_N)$
$\mathrm{SQN}_N \leftarrow \mathrm{SQN}_N + 1$
$\mathsf{b}_{\mathsf{mac}} \wedge \mathsf{b}_{\mathrm{SQN}}$
$\mathrm{SQN}_U \leftarrow \mathrm{SQN}_N$
$\mathsf{H}^2_\mathsf{k}(\mathsf{n})$
$\neg \mathsf{b}_{\mathsf{mac}}$
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If the mac is valid:
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4G-AKA

The IMSI Catcher Attack [Strobel, 2007]

No Confidentiality of the User Identity

The ID is sent in plain text!

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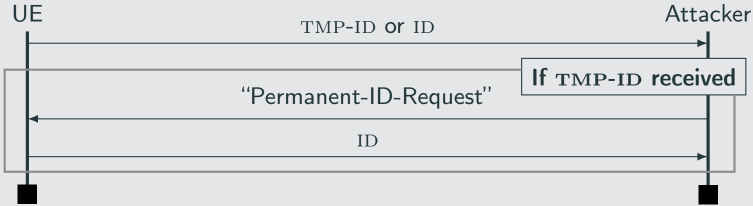

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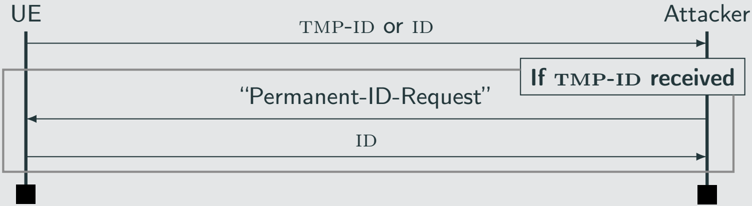

Why This is a Major Attack

- **Reliable**: always works.
- **Easy to deploy**: only needs an antenna.
- **Large scale**: is not targeted.

Privacy in 5G-AKA

The 5G-AKA protocol

5G-AKA is the next version of AKA (drafts are available).

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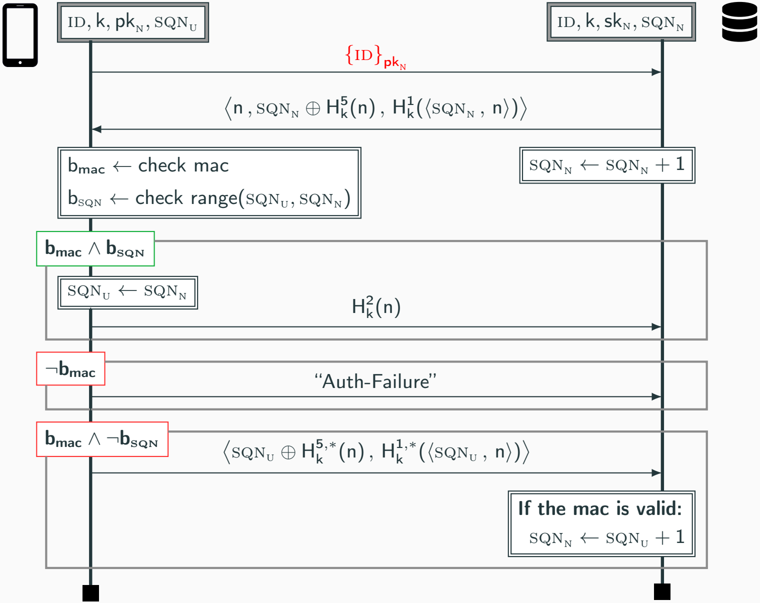
3GPP fix for 5G-AKA

Simply **encrypts** the permanent identity by sending $\{\mathrm{ID}\}_{\mathsf{pk}_{\mathrm{N}}}$



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$\{ID\}_{pk_N}$
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5G-AKA

Privacy in 5G-AKA

Is it enough?

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For confidentiality of the ID, yes.

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For confidentiality of the ID, yes.

For unlinkability, no.

Unlinkability

Unlinkability Attack

Even if ID is hidden, an attacker can **link sessions of a user**.

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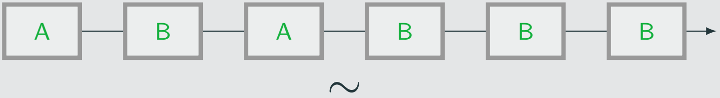

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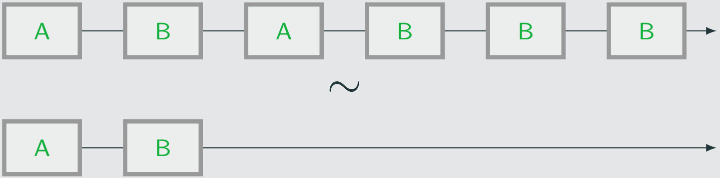

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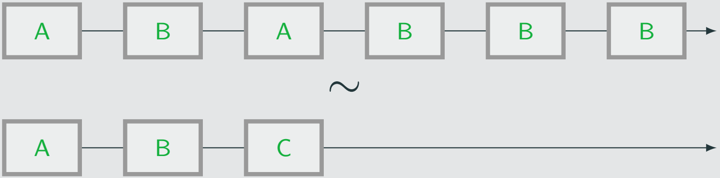

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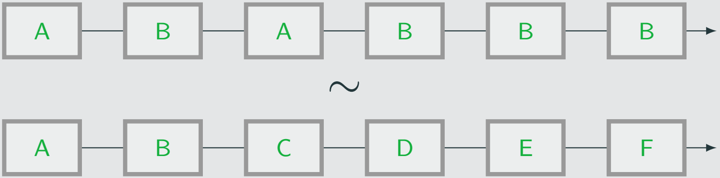
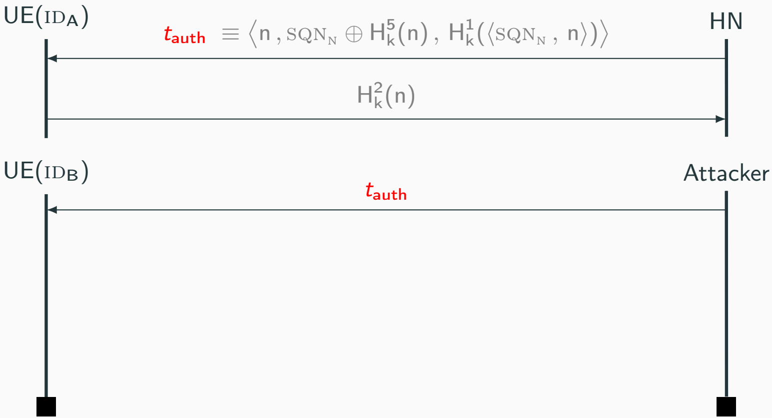


The Failure Message Attack [Arapinis et al., 2012]



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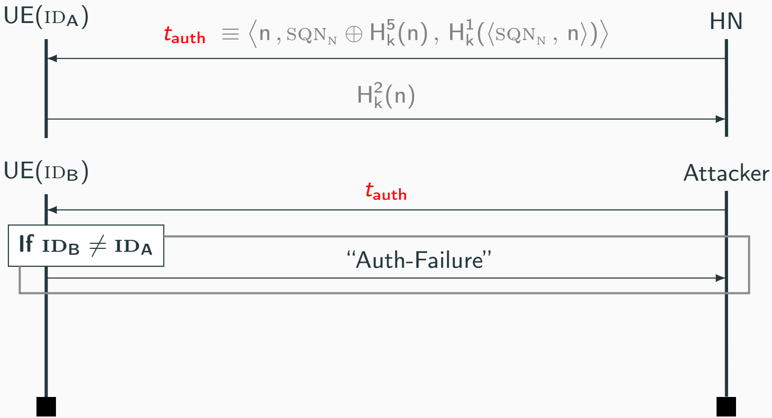

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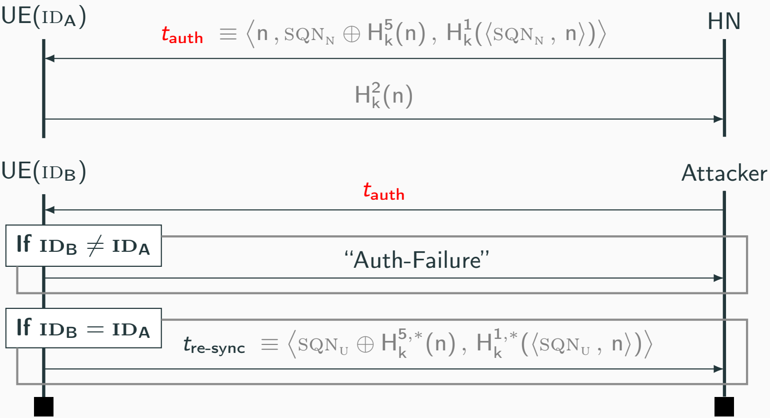

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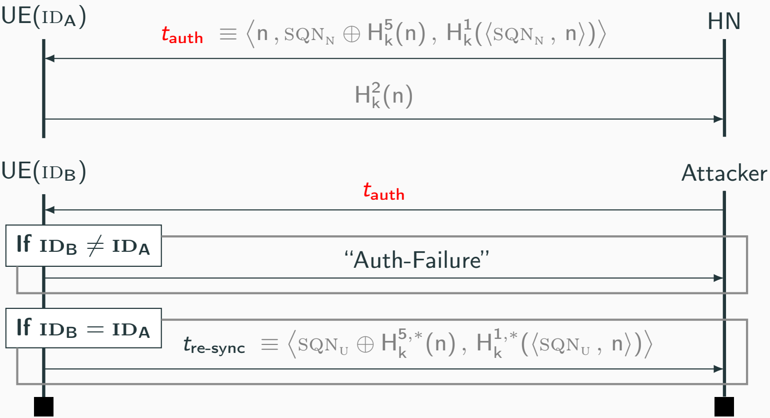

Unlinkability Attack

The adversary knows if it interacted with ID_A or ID_B.

Goal

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- Provides some form of **unlinkability**.

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- Is **proved secure**.

Theorem

Theorem

The AKA^+ protocol is σ-unlinkable for **an arbitrary number of agents and sessions** when:

- The asymmetric encryption $\{\_\}_{\_}$ is $\textsc{ind-cca}_1$.
- H and H^r (resp. Mac^1–Mac^5) are jointly $\textsc{prf}$.

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Remarks

- **Computational** security.
- AKA$^+$ is **stateful**, and uses the $\oplus$ **operator**.
- The proof is technical (around 80 pages).

The Bana-Comon Model

Example of a Protocol

A Simple Handshake

$$\begin{aligned} 1 : A \longrightarrow B &\quad : \quad n_A \\ 2 : B \longrightarrow A &\quad : \quad \{\langle B \, , \, n_A \rangle\}_{pk(A)} \end{aligned}$$

Bana-Comon Model: Messages

Messages

We use terms to model *protocol messages*, built upon:

- **Names** $\mathcal{N}$, e.g. $\mathsf{n_A}, \mathsf{n_B}$, for random samplings.
- **Function symbols** $\mathcal{F}$, e.g.:

$$\mathsf{A},\ \mathsf{B},\ \langle \_ , \_ \rangle,\ \pi_i(\_),\ \{\_\}_{\_},\ \mathsf{pk}(\_),\ \mathsf{sk}(\_)$$

$$\mathsf{if\_then\_else\_},\ \mathsf{eq}(\_, \_)$$

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Examples

$$\langle \mathsf{n_A}\,,\,\mathsf{A}\rangle \qquad \pi_1(\mathsf{n_B}) \qquad \{\langle \mathsf{B}\,,\,\mathsf{n_A}\rangle\}_{\mathsf{pk}(\mathsf{A})}$$

Bana-Comon Model: Messages

A Simple Handshake

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How do we represent the adversary's inputs?

Bana-Comon Model: Messages

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How do we represent the adversary's inputs?

- We use an **adversarial functions symbol g**.
  g's input is the current knowledge of the adversary.

Bana-Comon Model: Messages

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How do we represent the adversary's inputs?

- We use an **adversarial functions symbol g**.
  g's input is the current knowledge of the adversary.
- Intuitively, **g** can be any PPTM.

Bana-Comon Model: Messages

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$$2 : \mathsf{B} \longrightarrow \mathsf{A} \quad : \quad \{\langle \mathsf{B} , \boxed{\mathsf{n_A}} \rangle\}_{\mathsf{pk(A)}}$$

Term Representing the Messages

$$t_1 = \mathsf{n_A}$$

Bana-Comon Model: Messages

A Simple Handshake

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$$2 : \mathsf{B} \longrightarrow \mathsf{A} \quad : \quad \{\langle \mathsf{B}\,, \boxed{\mathsf{n_A}} \rangle\}_{\mathsf{pk(A)}}$$

Term Representing the Messages

$$t_1 = \mathsf{n_A}$$
$$t_2 = \left\{\left\langle \mathsf{B}, \boxed{\mathbf{g}(t_1)} \right\rangle\right\}_{\mathsf{pk(A)}}$$

Bana-Comon Model: Security Properties

Formula

Formulas are built using a predicate $\sim$ of arbitrary arity.

Bana-Comon Model: Security Properties

Formula

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Example

$$\mathsf{n} \quad \sim \quad \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'$$

Example of a Proof

$$\frac{}{\mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'}$$

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$$\frac{t \sim u}{s \sim u}\ \mathrm{R}$$

when $s =_R t$

($x =_R$ if b then x else x)

Example of a Proof

$$\frac{\text{if } g() \text{ then n else n} \sim \text{if } g() \text{ then n else n}'}{\text{n} \sim \text{if } g() \text{ then n else n}'}\ \mathsf{R}$$

$$\frac{t \sim u}{s \sim u}\ \mathsf{R}$$

when $s =_R t$

$(x =_R \text{ if } b \text{ then } x \text{ else } x)$

Example of a Proof

$$\frac{\text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'}{\mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'} \text{ R}$$

$$\frac{t \sim u}{s \sim u} \text{ R}$$

when $s =_R t$

$(x =_R \text{if } b \text{ then } x \text{ else } x)$

$$\frac{b, u \sim b', u' \qquad b, v \sim b', v'}{\text{if } b \text{ then } u \text{ else } v \sim \text{if } b' \text{ then } u' \text{ else } v'} \text{ CS}$$

Example of a Proof

$$\dfrac{\dfrac{}{g(), \mathsf{n} \sim g(), \mathsf{n}} \qquad \dfrac{}{g(), \mathsf{n} \sim g(), \mathsf{n}'}}{\dfrac{\text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'}{\mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'} \text{ R}} \text{ CS}$$

$$\frac{t \sim u}{s \sim u} \text{ R}$$

when $s =_R t$

$(x =_R \text{if } b \text{ then } x \text{ else } x)$

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Example of a Proof

$$\dfrac{\dfrac{}{g(), \mathsf{n} \sim g(), \mathsf{n}}\text{ Refl} \qquad \dfrac{}{g(), \mathsf{n} \sim g(), \mathsf{n}'}\text{ Refl}}{\dfrac{\text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'}{\mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'}\text{ R}}\text{ CS}$$

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Decision Result

Decision Problem: Derivability

Input: A ground formula $\vec{u} \sim \vec{v}$.

Question: Is there a derivation of $\vec{u} \sim \vec{v}$ using Ax?

Decidability

Decision Problem: Derivability

Input: A ground formula $\vec{u} \sim \vec{v}$.

Question: Is there a derivation of $\vec{u} \sim \vec{v}$ using Ax?

or equivalently

Decision Problem: Game Transformations

Input: A game $\vec{u} \sim \vec{v}$.

Question: Is there a sequence of cryptographic game transformations in Ax showing that $\vec{u} \sim \vec{v}$ is secure?

The Set of Axioms Ax

$$\frac{u \sim t}{u \sim s}\ \mathsf{R}$$

when $s =_R t$

$$\frac{b, u \sim b', u' \qquad b, v \sim b', v'}{\text{if } b \text{ then } u \text{ else } v \sim \text{if } b' \text{ then } u' \text{ else } v'}\ \mathsf{CS}$$

The Set of Axioms Ax

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$$\frac{b, u \sim b', u' \qquad b, v \sim b', v'}{\text{if } b \text{ then } u \text{ else } v \sim \text{if } b' \text{ then } u' \text{ else } v'}\ \mathrm{CS}$$

$$\frac{x \sim y}{x, x \sim y, y}\ \mathrm{Dup}$$

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$$\frac{x \sim y}{x, x \sim y, y}\ \mathsf{Dup}$$

$$\frac{x_1, \ldots, x_n \sim y_1, \ldots, y_n}{f(x_1, \ldots, x_n) \sim f(y_1, \ldots, y_n)}\ \mathsf{FA}$$

The Set of Axioms Ax

$$\frac{u \sim t}{u \sim s}\ \mathrm{R}$$

when $s =_R t$

$$\frac{b, u \sim b', u' \qquad b, v \sim b', v'}{\text{if } b \text{ then } u \text{ else } v \sim \text{if } b' \text{ then } u' \text{ else } v'}\ \mathrm{CS}$$

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$$\frac{}{\vec{u}, \{s\}_{\mathsf{pk}(\mathsf{n})} \sim \vec{u}, \{t\}_{\mathsf{pk}(\mathsf{n})}}\ \mathrm{CCA1}$$

when ...

Equational Theory

Equational Theory: Protocol Functions

- $\pi_i(\langle x_1, x_2 \rangle) = x_i$ $\qquad i \in \{1, 2\}$
- $\mathsf{dec}(\{x\}_{\mathsf{pk}(\mathsf{y})}, \mathsf{sk}(\mathsf{y})) = x$

Equational Theory

Equational Theory: Protocol Functions

If Homomorphism:

$$f(\vec{u}, \text{if } b \text{ then } x \text{ else } y, \vec{v}) = \text{if } b \text{ then } f(\vec{u}, x, \vec{v}) \text{ else } f(\vec{u}, y, \vec{v})$$

$$\text{if } (\text{if } b \text{ then } a \text{ else } c) \text{ then } x \text{ else } y = \text{if } b \text{ then } (\text{if } a \text{ then } x \text{ else } y) \text{ else } (\text{if } c \text{ then } x \text{ else } y)$$

If Rewriting:

$$\text{if } b \text{ then } x \text{ else } x = x$$

$$\text{if } b \text{ then } (\text{if } b \text{ then } x \text{ else } y) \text{ else } z = \text{if } b \text{ then } x \text{ else } z$$

$$\text{if } b \text{ then } x \text{ else } (\text{if } b \text{ then } y \text{ else } z) = \text{if } b \text{ then } x \text{ else } z$$

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$$\text{if } b \text{ then } (\text{if } a \text{ then } x \text{ else } y) \text{ else } z = \text{if } a \text{ then } (\text{if } b \text{ then } x \text{ else } z) \text{ else } (\text{if } b \text{ then } y \text{ else } z)$$

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Equational Theory: Protocol Functions

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If Rewriting:

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If Re-Ordering:

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Strategy

Deconstructing Rules

Rules CCA1, CS, FA and Dup are decreasing transformations.

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$$\frac{x \sim y}{x, x \sim y, y}\ \mathrm{Dup}$$

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$$\frac{}{\vec{u}, \{s\}_{\mathrm{pk}(n)} \sim \vec{u}, \{t\}_{\mathrm{pk}(n)}}\ \mathrm{CCA1}$$

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when ...

Problem

The rule R is not decreasing!

Difficulties

If Introduction: $x \to$ **if** b **then** x **else** x

$$\dfrac{\dfrac{}{g(), \mathsf{n} \sim g(), \mathsf{n}}\ \text{Refl} \qquad \dfrac{}{g(), \mathsf{n} \sim g(), \mathsf{n}'}\ \text{Refl}}{\dfrac{\text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'}{\mathsf{n} \sim \text{if } g() \text{ then } \mathsf{n} \text{ else } \mathsf{n}'}\ \text{R}}\ \text{CS}$$

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Bounded Introduction

The introduced conditional $g()$ is bounded by the other side.

Decision Procedure

Proof Cut: Introduction of a Conditional on Both Sides

$$\dfrac{\dfrac{a, s \sim b, t \qquad\qquad a, s \sim b, t}{\text{if } a \text{ then } s \text{ else } s \sim \text{if } b \text{ then } t \text{ else } t}\ \text{CS}}{s \sim t}\ \text{R}$$

Decision Procedure

Proof Cut: Introduction of a Conditional on Both Sides

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Lemma

We can **extract** from $a, s \sim b, t$ a (smaller) proof of $s \sim t$.

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Proof Cut: Introduction of a Conditional on Both Sides

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Lemma

We can **extract** from $a, s \sim b, t$ a (smaller) proof of $s \sim t$.

$\Rightarrow$ **Proof Cut Elimination**

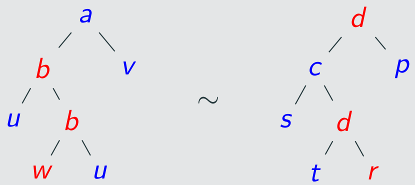
Decision Procedure

Proof Cut

$$\frac{}{\text{if } a \text{ then } u \text{ else } v \sim \text{if } c \text{ then } s \text{ else } t}$$

Decision Procedure

Proof Cut



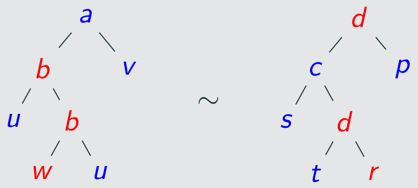
$$\frac{\qquad}{\text{if } a \text{ then } u \text{ else } v \sim \text{if } c \text{ then } s \text{ else } t}\ \text{R}$$

where $p \equiv \text{if } c \text{ then } s \text{ else } t$

Decision Procedure

Proof Cut

$$\dfrac{a, b, b, u, w, u, v \sim d, c, d, s, t, r, p}{}\ \text{FA}^{(3)}$$



$$\dfrac{}{\text{if } a \text{ then } u \text{ else } v \sim \text{if } c \text{ then } s \text{ else } t}\ \text{R}$$

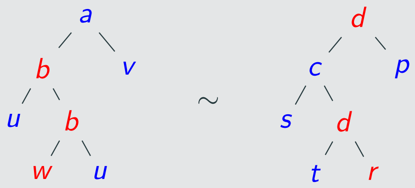
where $p \equiv \text{if } c \text{ then } s \text{ else } t$

Decision Procedure

Proof Cut

$$a, b, b, u, w, u, v \sim d, c, d, s, t, r, p$$

FA$^{(3)}$



R

$$\text{if } a \text{ then } u \text{ else } v \sim \text{if } c \text{ then } s \text{ else } t$$

where $p \equiv \text{if } c \text{ then } s \text{ else } t$

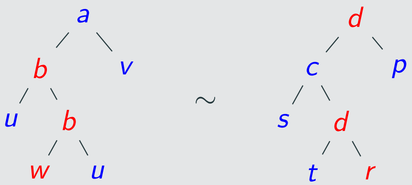
Key Lemma

If $b, b \sim c, d$ can be shown using only FA, Dup and CCA1 then:

$$c \equiv d$$

Proof Cut

$$\frac{a, b, b, u, w, u, v \sim d, c, d, s, t, r, p}{}\ \text{FA}^{(3)}$$



$$\frac{}{\text{if } a \text{ then } u \text{ else } v \sim \text{if } c \text{ then } s \text{ else } t}\ \text{R}$$

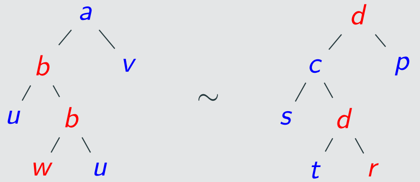
where $p \equiv \text{if } c \text{ then } s \text{ else } t$

Proof Cut Elimination

- $b, b \sim c, d \quad \Longrightarrow \quad c \equiv d.$

Proof Cut

$$a, b, b, u, w, u, v \sim d, c, d, s, t, r, p \quad \text{FA}^{(3)}$$



$$\text{if } a \text{ then } u \text{ else } v \sim \text{if } c \text{ then } s \text{ else } t \quad \text{R}$$

where $p \equiv$ if c then s else t

Proof Cut Elimination

- $b, b \sim c, d \quad \Longrightarrow \quad c \equiv d$.
- $a, b \sim d, c \quad \Longrightarrow \quad a \equiv b$.

Strategy: Theorem

Theorem

The following problem is decidable:

Input: A ground formula $\vec{u} \sim \vec{v}$.

Question: Is there a derivation of $\vec{u} \sim \vec{v}$ using Ax?

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This holds when using CCA2 as unitary inference rules.

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This holds when using CCA2 as unitary inference rules.

Sketch

- Commute rule applications to order them as follows:

$$(\mathsf{2Box} + R_{\square}) \cdot \mathsf{CS}_{\square} \cdot \mathsf{FA}_{\mathsf{if}} \cdot \mathsf{FA}_{\mathsf{f}} \cdot \mathsf{Dup} \cdot \mathsf{CCA2}$$

- We do proof cut eliminations to get a small proof.

Conclusion

Conclusion: Contributions

RFID Protocols

Studied the privacy of two RFID protocols, KCL and LAK.

The 5G-AKA Protocol

- Showed that some attacks against 4G-AKA apply to 5G-AKA.
- Proposed a fixed version, and proved it secure in the computational model.
- Found a new privacy attack on another protocol, PRIV-AKA.

Conclusion: Contributions

Decidability Result

- Decidability of a set of inference rules for computational indistinguishability.
- First decidability result for a non-trivial set of cryptographic game transformations.

Perspectives

Study the Scope of the Decidability Result

- Support for a larger class of primitives and associated assumptions.
- Undecidability results for extensions of the set of axioms.

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Study the Scope of the Decidability Result

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Proof Automation for the AKA$^+$ Case Study

- AKA$^+$ security proof is very lengthy (around 80 pages).
- The proofs are out-of-scope of the decidability result:
  - Arbitrary number of sessions (induction).
  - Reasoning on sequence numbers.

$\Rightarrow$ We need some proof automation/mechanization.

References i

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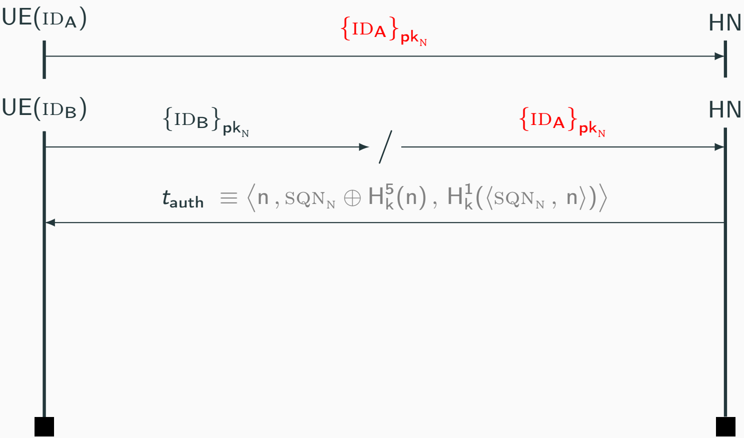
The Encrypted ID Replay Attack



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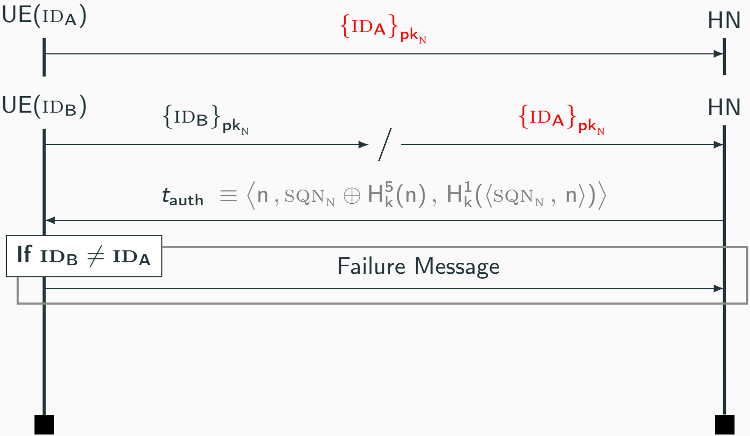

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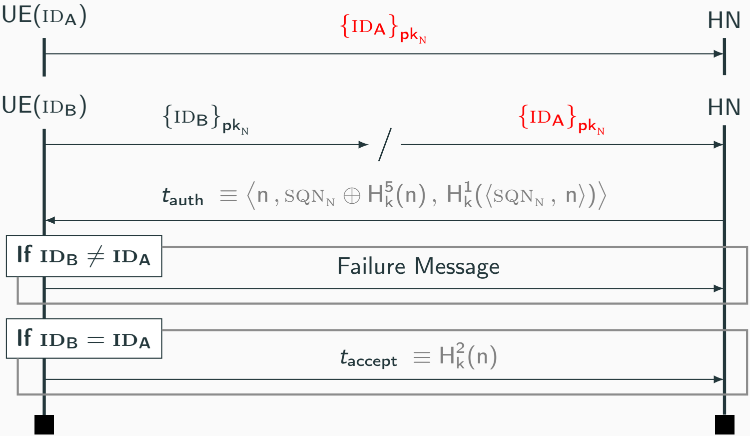
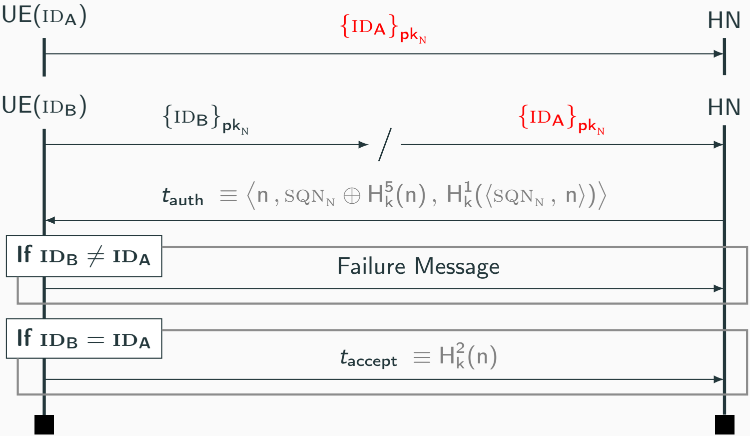


The Encrypted ID Replay Attack



Unlinkability Attack

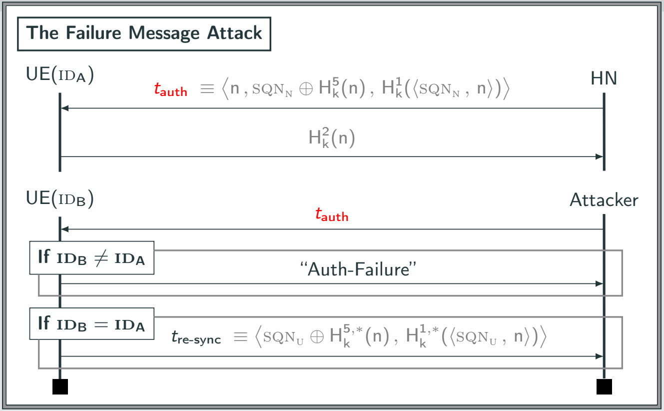
The adversary knows if it interacted with ID_A or ID_B.

Key Ideas

Key Ideas Behind AKA$^+$

Key Ideas

Key Ideas Behind AKA^{+}



Key Ideas

Key Ideas Behind AKA^+

- **Postpone re-synchronization** to the next session:

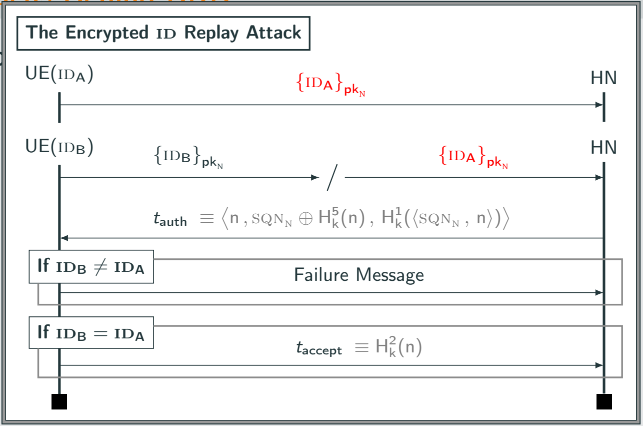
$$\{\langle \mathrm{ID}\, ,\, \mathrm{SQN_U} \rangle\}_{\mathsf{pk_N}}$$

  - No re-synchronization message $\Longrightarrow$ no failure message attack.
  - No extra randomness for the user.

Key Ideas

Key Ideas Behind AKA$^+$

- Po… attack.



Key Ideas

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$$\{\langle \mathrm{ID}\ ,\ \mathrm{SQN_U} \rangle\}_{\mathsf{pk_N}}$$

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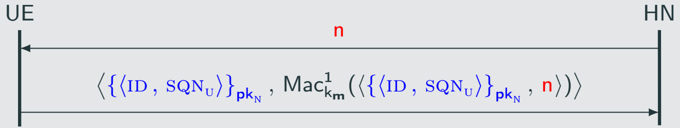
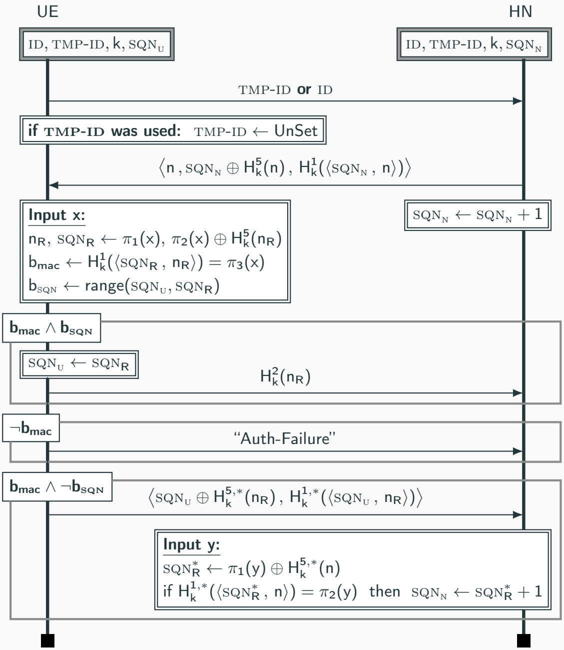
- Add a challenge n from the HN when using the **permanent identity**.

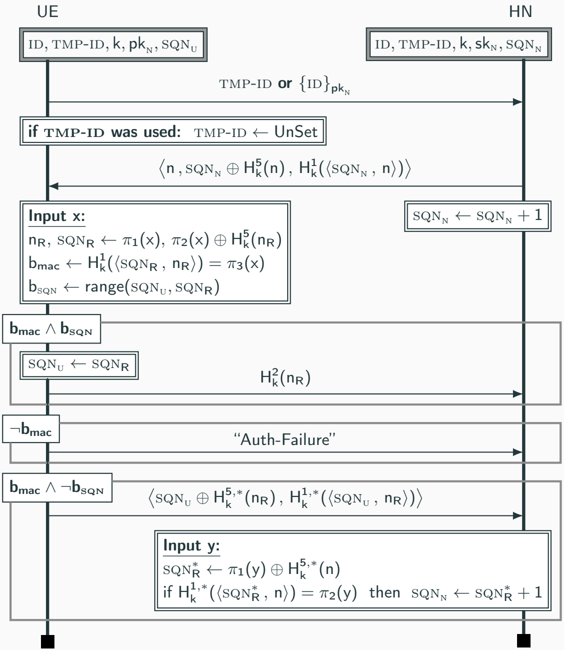


4G-AKA

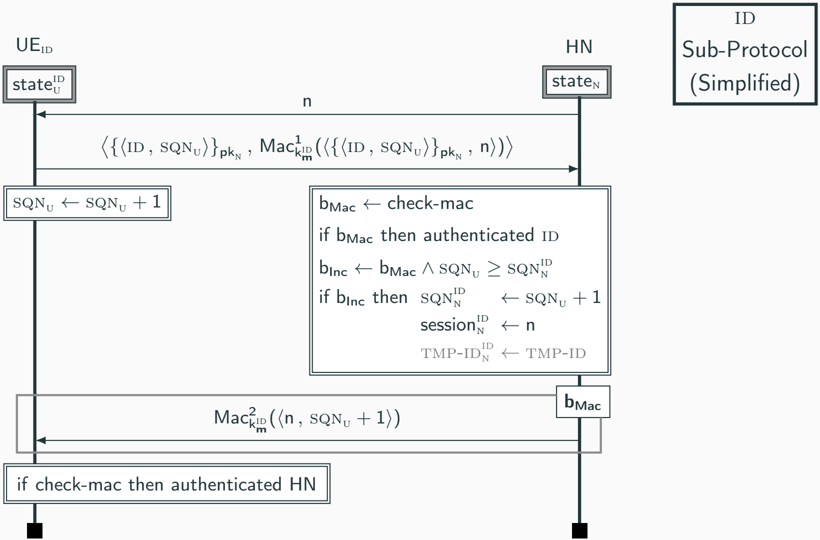


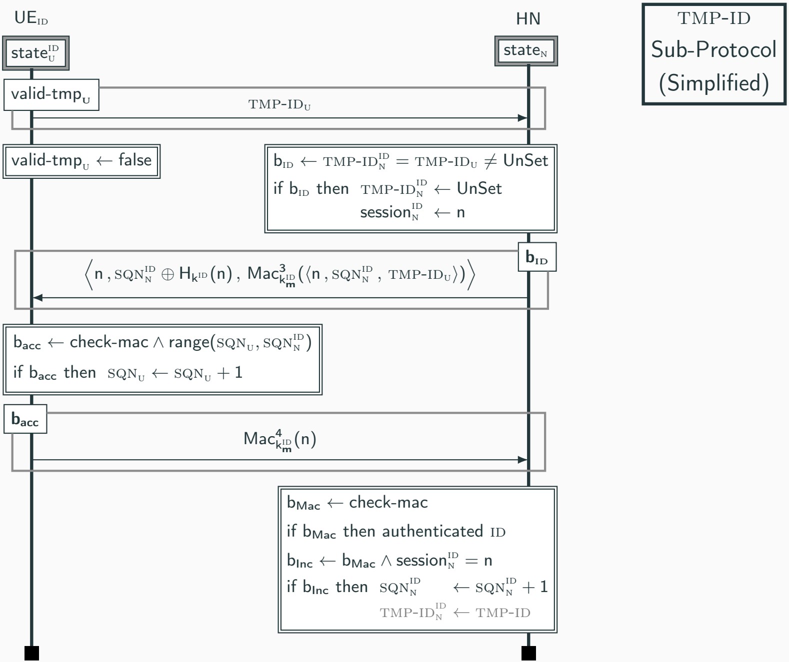
UE
HN
$\mathrm{ID}, \mathrm{TMP\text{-}ID}, k, \mathrm{SQN}_U$
$\mathrm{ID}, \mathrm{TMP\text{-}ID}, k, \mathrm{SQN}_N$
TMP-ID or ID
if TMP-ID was used: TMP-ID $\leftarrow$ UnSet
$\langle n, \mathrm{SQN}_N \oplus H^5_k(n), H^1_k(\langle \mathrm{SQN}_N, n\rangle)\rangle$
$\mathrm{SQN}_N \leftarrow \mathrm{SQN}_N + 1$
Input x:
$n_R, \mathrm{SQN}_R \leftarrow \pi_1(x), \pi_2(x) \oplus H^5_k(n_R)$
$b_{mac} \leftarrow H^1_k(\langle \mathrm{SQN}_R, n_R\rangle) = \pi_3(x)$
$b_{SQN} \leftarrow \mathrm{range}(\mathrm{SQN}_U, \mathrm{SQN}_R)$
$b_{mac} \wedge b_{SQN}$
$\mathrm{SQN}_U \leftarrow \mathrm{SQN}_R$
$H^2_k(n_R)$
$\neg b_{mac}$
"Auth-Failure"
$b_{mac} \wedge \neg b_{SQN}$
$\langle \mathrm{SQN}_U \oplus H^{5,*}_k(n_R), H^{1,*}_k(\langle \mathrm{SQN}_U, n_R\rangle)\rangle$
Input y:
$\mathrm{SQN}^*_R \leftarrow \pi_1(y) \oplus H^{5,*}_k(n)$
if $H^{1,*}_k(\langle \mathrm{SQN}^*_R, n\rangle) = \pi_2(y)$ then $\mathrm{SQN}_N \leftarrow \mathrm{SQN}^*_R + 1$

5G-AKA

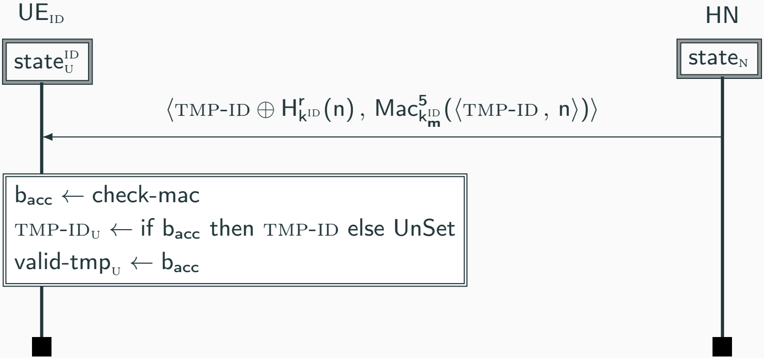


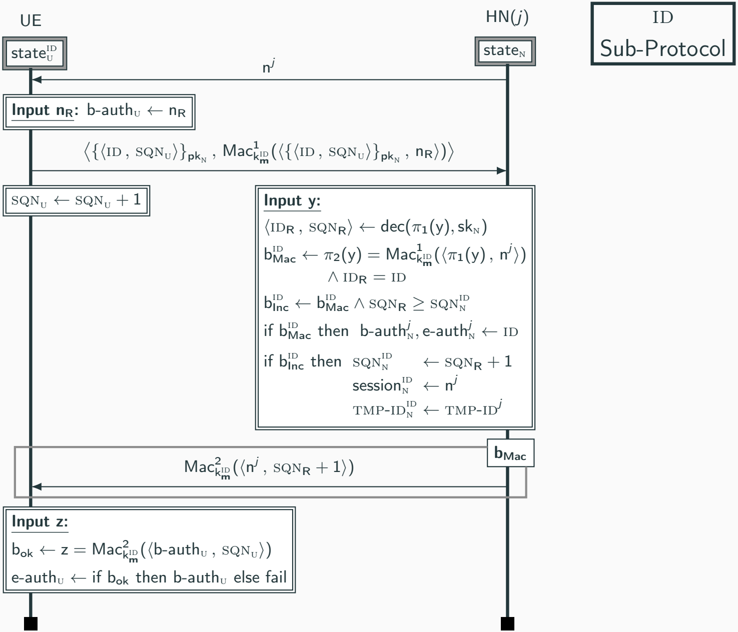
UE
HN
ID, TMP-ID, k, pk_N, SQN_U
ID, TMP-ID, k, sk_N, SQN_N
TMP-ID or $\{ID\}_{pk_N}$
if TMP-ID was used: TMP-ID ← UnSet
$\langle n, SQN_N \oplus H^5_k(n), H^1_k(\langle SQN_N, n\rangle)\rangle$
$SQN_N \leftarrow SQN_N + 1$
Input x:
$n_R, SQN_R \leftarrow \pi_1(x), \pi_2(x) \oplus H^5_k(n_R)$
$b_{mac} \leftarrow H^1_k(\langle SQN_R, n_R\rangle) = \pi_3(x)$
$b_{SQN} \leftarrow range(SQN_U, SQN_R)$
$b_{mac} \wedge b_{SQN}$
$SQN_U \leftarrow SQN_R$
$H^2_k(n_R)$
$\neg b_{mac}$
"Auth-Failure"
$b_{mac} \wedge \neg b_{SQN}$
$\langle SQN_U \oplus H^{5,*}_k(n_R), H^{1,*}_k(\langle SQN_U, n_R\rangle)\rangle$
Input y:
$SQN^*_R \leftarrow \pi_1(y) \oplus H^{5,*}_k(n)$
if $H^{1,*}_k(\langle SQN^*_R, n\rangle) = \pi_2(y)$ then $SQN_N \leftarrow SQN^*_R + 1$


ID
Sub-Protocol
(Simplified)
UE_ID
HN
state_U^ID
state_N
n
⟨{⟨ID , SQN_U⟩}_pk_N , Mac^1_k_m^ID(⟨{⟨ID , SQN_U⟩}_pk_N , n⟩)⟩
SQN_U ← SQN_U + 1
b_Mac ← check-mac
if b_Mac then authenticated ID
b_Inc ← b_Mac ∧ SQN_U ≥ SQN_N^ID
if b_Inc then SQN_N^ID ← SQN_U + 1
session_N^ID ← n
TMP-ID_N^ID ← TMP-ID
b_Mac
Mac^2_k_m^ID(⟨n , SQN_U + 1⟩)
if check-mac then authenticated HN

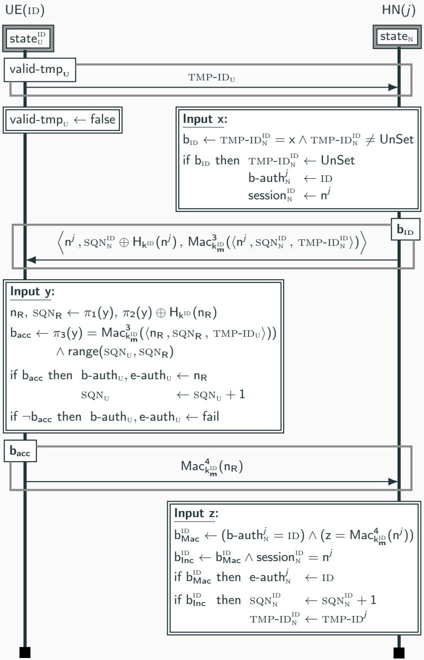

UE_ID
HN
TMP-ID Sub-Protocol (Simplified)
state_U^ID
state_N
valid-tmp_U
TMP-ID_U
valid-tmp_U ← false
b_ID ← TMP-ID_N^ID = TMP-ID_U ≠ UnSet
if b_ID then TMP-ID_N^ID ← UnSet
session_N^ID ← n
b_ID
⟨n, SQN_N^ID ⊕ H_k^ID(n), Mac^3_{k_m^ID}(⟨n, SQN_N^ID, TMP-ID_U⟩)⟩
b_acc ← check-mac ∧ range(SQN_U, SQN_N^ID)
if b_acc then SQN_U ← SQN_U + 1
b_acc
Mac^4_{k_m^ID}(n)
b_Mac ← check-mac
if b_Mac then authenticated ID
b_Inc ← b_Mac ∧ session_N^ID = n
if b_Inc then SQN_N^ID ← SQN_N^ID + 1
TMP-ID_N^ID ← TMP-ID

The ASSIGN-TMP-ID Sub-Protocol (Simplified)

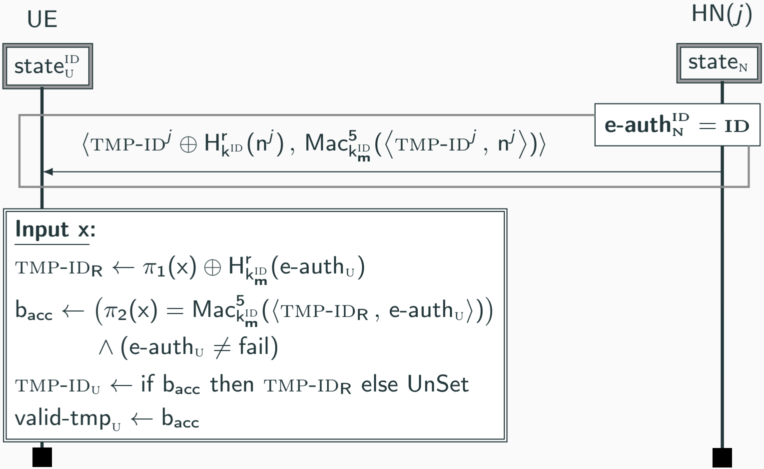



UE
HN(j)
ID Sub-Protocol
state_U^{ID}
state_N
n^j
Input $\mathsf{n_R}$: $\mathsf{b\text{-}auth}_U \leftarrow \mathsf{n_R}$
$\langle \{\langle ID , SQN_U \rangle\}_{\mathsf{pk_N}} , \mathsf{Mac}^1_{\mathsf{k_m^{ID}}}(\langle \{\langle ID , SQN_U \rangle\}_{\mathsf{pk_N}} , \mathsf{n_R} \rangle) \rangle$
$SQN_U \leftarrow SQN_U + 1$
Input y:
$\langle ID_R , SQN_R \rangle \leftarrow \mathsf{dec}(\pi_1(\mathsf{y}), \mathsf{sk_N})$
$\mathsf{b}^{ID}_{\mathsf{Mac}} \leftarrow \pi_2(\mathsf{y}) = \mathsf{Mac}^1_{\mathsf{k_m^{ID}}}(\langle \pi_1(\mathsf{y}) , \mathsf{n}^j \rangle)$
$\wedge\ ID_R = ID$
$\mathsf{b}^{ID}_{\mathsf{Inc}} \leftarrow \mathsf{b}^{ID}_{\mathsf{Mac}} \wedge SQN_R \geq SQN_N^{ID}$
if $\mathsf{b}^{ID}_{\mathsf{Mac}}$ then $\mathsf{b\text{-}auth}_N^j, \mathsf{e\text{-}auth}_N^j \leftarrow ID$
if $\mathsf{b}^{ID}_{\mathsf{Inc}}$ then $SQN_N^{ID} \leftarrow SQN_R + 1$
$\mathsf{session}_N^{ID} \leftarrow \mathsf{n}^j$
$TMP\text{-}ID_N^{ID} \leftarrow TMP\text{-}ID^j$
$\mathsf{b_{Mac}}$
$\mathsf{Mac}^2_{\mathsf{k_m^{ID}}}(\langle \mathsf{n}^j , SQN_R + 1 \rangle)$
Input z:
$\mathsf{b_{ok}} \leftarrow \mathsf{z} = \mathsf{Mac}^2_{\mathsf{k_m^{ID}}}(\langle \mathsf{b\text{-}auth}_U , SQN_U \rangle)$
$\mathsf{e\text{-}auth}_U \leftarrow$ if $\mathsf{b_{ok}}$ then $\mathsf{b\text{-}auth}_U$ else fail

TMP-ID
Sub-Protocol



The ASSIGN-TMP-ID Sub-Protocol



New Attack on the PRIV-AKA Protocol

The PRIV-AKA Protocol

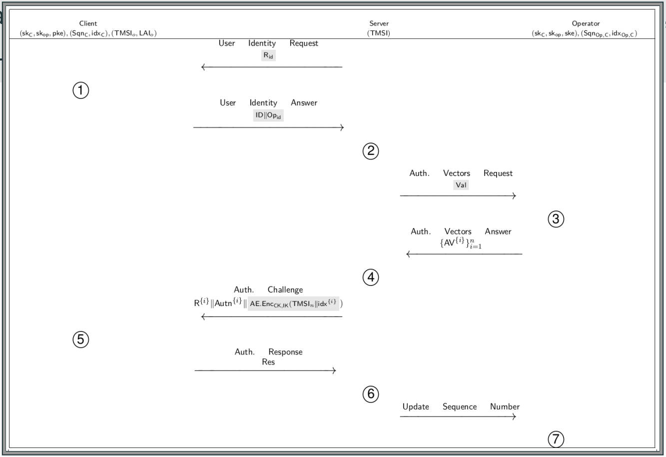
The authors of [Fouque et al., 2016] propose a new protocol, PRIV-AKA (claimed unlinkable).

New Attack on the PRIV-AKA Protocol

The PRIV-AKA Protocol

The a ... ,

PRIV-



New Attack on the PRIV-AKA Protocol

The PRIV-AKA Protocol

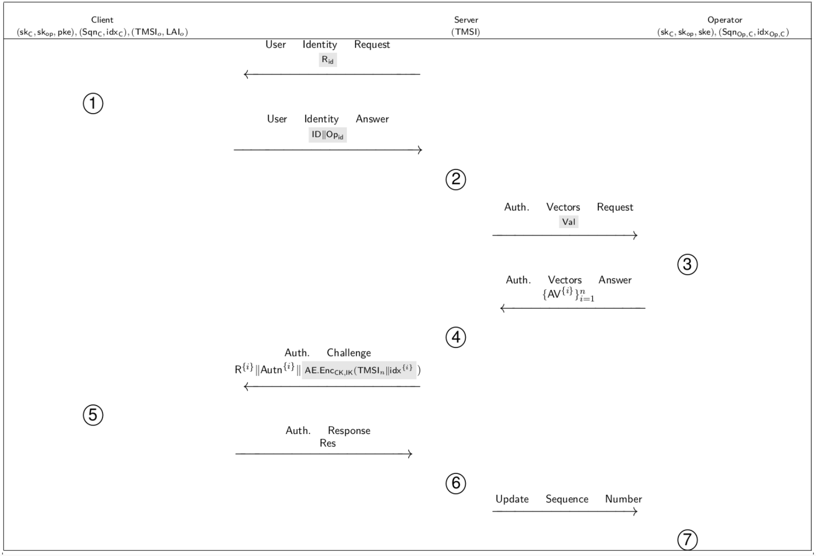
The authors of [Fouque et al., 2016] propose a new protocol, PRIV-AKA (claimed unlinkable).

Unlinkability Attack (four sessions)

We found an attack to permanently de-synchronize the user:

- Run a session but keep the last message t_1.
- Re-synchronize the user and the network.
- Re-iterate the last two steps to get a second message t_2.
- Re-synchronize the user and the network.
- Send both t_1 and t_2, which increments SQN_N by **two**.
- The user is permanently de-synchronized
  $\Longrightarrow$ unlinkability attack.

PRIV-AKA [Fouque et al., 2016]



PRIV-AKA [Fouque et al., 2016]

Client

①: Compute the identifier:
If $\text{flag}_{\text{TMSI}} := 0$ then ID = TMSI.
Else, $\text{ID} = \text{PKE.Enc}_{\text{pke}}(f_5(\text{keys}, \text{R}_{\text{id}}, \text{IMSI}, \text{idx}_{\text{C}}) \| \text{R}_{\text{id}} \| \text{IMSI} \| \text{idx}_{\text{C}})$.
$\text{flag}_{\text{TMSI}} := 1$.

⑤: Compute AK using $\text{R}^{\{i\}}$.
Recover $\text{Sqn}^{\{i\}}$ (from AK).
Check Mac_{S} value.
Compute: IK, CK;
Retrieve the received index and the new TMSI.
If abort caused or the AE does not verify, set $\text{flag}_{\text{TMSI}} := 1$ and increment: $\text{idx}_{\text{C}} := \text{idx}_{\text{C}} + 1$.
Else, check validity of $\text{Sqn}^{\{i\}}$, i.e if one of the following conditions is correct:
- $\text{Sqn}_{\text{C}} = \text{Sqn}^{\{i\}}$.
- $\text{Sqn}_{\text{C}} = \text{inc}(\text{Sqn}^{\{i\}})$ and $\text{idx}^{\{i\}} = \text{idx}_{\text{C}} + 1$.

If the first condition is accepted: reset the index idx_{C}, update the sequence number $\text{Sqn}_{\text{C}} = \text{inc}(\text{Sqn}_{\text{C}})$.
If the second condition is accepted: idx_{C}=idx_{C}+1 .

Compute $\text{Res} := \mathcal{F}_1^*(\text{keys}, \text{R}^{\{i\}}, \text{Sqn}^{\{i\}}, \text{Res}_{\text{S}}, \text{AMF})$.
Update the internal index. Allocate the new TMSI.
$\text{flag}_{\text{TMSI}} := 0$.

Server

②: Process the identifier ID:
If the identifier is a TMSI then Val = IMSI. Otherwise, $\text{Val} = (\text{ID}, \text{R}_{\text{id}})$.

④: Store $\{\text{AV}^{\{i\}}\}_{i=1}^{n}$.
Choose $\text{AV}^{\{i\}}$ one by one in order. Then, it sends the authentication challenge and the new couple $(\text{TMSI}_n, \text{idx}^{\{i\}})$ encrypted and authenticated by the session keys.

⑥: If the authentication of the client is verified ($\text{Res} \stackrel{?}{=} \text{Mac}_{\text{C}}$), then they ask to the server the update of its sequence number. Otherwise, the protocol is aborted.

Operator

③: Verify the identity of the client with Val.

If this holds, retrieve idx_{C}, set $\text{idx}_{\text{Op,C}} := \text{idx}_{\text{C}}$
Generate $(\text{R}^{\{1\}}, ..., \text{R}^{\{n\}})$. Denote: $\text{keys} := (\text{sk}_{\text{C}}, \text{sk}_{\text{op}})$.
For each $i = 1, \ldots, n$, compute:
$\text{Mac}_{\text{S}} \leftarrow \mathcal{F}_1(\text{keys}, \text{R}^{\{i\}}, \text{Sqn}^{\{i\}}, \text{Res}_{\text{S}}, \text{AMF})$,
$\text{Mac}_{\text{C}} \leftarrow \mathcal{F}_1^*(\text{keys}, \text{R}^{\{i\}}, \text{Sqn}^{\{i\}}, \text{Res}_{\text{S}}, \text{AMF})$,
$\text{CK} \leftarrow \mathcal{F}_3(\text{keys}, \text{R}^{\{i\}}, \text{Sqn}^{\{i\}}, \text{Res}_{\text{S}}, \text{AMF})$,
$\text{IK} \leftarrow \mathcal{F}_4(\text{keys}, \text{R}^{\{i\}}, \text{Sqn}^{\{i\}}, \text{Res}_{\text{S}}, \text{AMF})$,
$\text{AK} \leftarrow \mathcal{F}_5(\text{keys}, \text{R}^{\{i\}}, \text{Res}_{\text{S}})$,
$\text{Autn}^{\{i\}} \leftarrow (\text{Sqn}^{\{i\}} \oplus \text{AK}) \| \text{AMF} \| \text{Mac}_{\text{S}}$,
$\text{Sqn}^{\{i\}} \leftarrow \text{inc}(\text{Sqn}^{\{i-1\}})$,
$\text{AV}^{\{i\}} := (\text{R}^{\{i\}}, \text{CK}, \text{IK}, \text{Autn}^{\{i\}}, \text{Mac}_{\text{C}}, \text{idx}^{\{i\}})$, with $\text{Sqn}^{\{1\}} := \text{Sqn}_{\text{Op,C}}$,
$\text{idx}^{\{1\}} := \text{idx}_{\text{Op,C}}$, $\forall i \neq 1, \text{idx}^{\{i\}} = 0$.
End for.

⑦ : Update the sequence number:
$\text{Sqn}_{\text{Op,C}} \leftarrow \text{inc}(\text{Sqn}_{\text{Op,C}})$. Reset the index $\text{idx}_{\text{Op,C}}$.

Counter-Examples

Remark: $\sim$ is not a congruence

Counter-Example: $\mathsf{n} \sim \mathsf{n}$ and $\mathsf{n} \sim \mathsf{n}'$, but $\mathsf{n}, \mathsf{n} \not\sim \mathsf{n}, \mathsf{n}'$.

Counter-Examples

Remark: $\sim$ is not a congruence

Counter-Example: $\mathsf{n} \sim \mathsf{n}$ and $\mathsf{n} \sim \mathsf{n}'$, but $\mathsf{n}, \mathsf{n} \not\sim \mathsf{n}, \mathsf{n}'$.

Congruence

If $\mathrm{eq}(u, v) \sim$ true then u and v are (almost always) *equal*

$\Rightarrow$ we have a congruence.

Counter-Examples

Remark: b is necessary in CS

$$\frac{b, u \sim b', u' \qquad b, v \sim b', v'}{\text{if } b \text{ then } u \text{ else } v \sim \text{if } b' \text{ then } u' \text{ else } v'}\ \text{CS}$$

We have:

$$\text{zero} \sim \text{zero} \qquad\qquad \text{one} \sim \text{one}$$

But:

$$\text{if true then zero else one} \not\sim \text{if false then zero else one}$$