Finitary decision structures in infinite games

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Abstract

In this paper we introduce a concept, called power matrix, which allows us to transform infinite-duration games into finite-duration ones. As a first step we observe that there exist infinite-duration games, even zero-sum ones, with infinitely many strategies which are not outcome equivalent. This constatation, which is independent of the objectives of players, implies that it is impossible to preserve all the information about outcomes while transforming an infinite-duration game into a finite-duration one. Our concept of power matrix uses imperfect information and non-determinism to allow a transformation which avoids as much as possible the loss of information. After introducing the concept, which is the main part of our study, we also give some applications showing that the power matrix is useful.

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Introduction

One crucial aspect in the analysis of reactive systems consists in capturing infinite behaviour. A typical requirement for such a system is that it never breaks down. However, physical computation devices can take into account only a finite amount of data. An effective approach for coping with ongoing interaction in discrete reactive systems is based on the theory of infinite games over finite graphs. The question of whether a given system can reach a designated configuration is just a problem of reachability in a graph if the system is closed, because such a system can be represented as a state machine. But if the system is open, i.e., if the environment can interact with it, then the problem can be modeled as a two-player game: one player is the system and the other the environment as explained in [4]. Another example of usefulness of game theory is the model checking problem, that is, the problem to check whether a given formula holds in a given model. Solving this problem corresponds to finding whether a player has a winning strategy in a given game. Kaiser gives more information about this approach in his thesis [3]. There are also some examples in [9]. As explained in [9], game theory can also be useful for the synthesis problem, i.e., the problem to construct a system with a given specification.

Originally, the games studied in computer science were only zero-sum games. In these games each player has to make the others lose in order to win. These games are now very well known but they are not sufficient to represent all realistic situations. For example, in the problem for an open system to reach a given configuration we can't assume that the environment is an opponent: the environment could be, for example, other systems which have the same goal. More generally in all games that are not zero-sum, the players could cooperate: they can win together. As soon as we allow players to cooperate (or even to lose intentionally) the theory becomes more complicated: we are not able to do predictions about the way the players will play. The case of finite-duration games can be solved, see [1], but the case of infinite-duration games is more complicated.

In this paper we focus on games of infinite duration. Our goal is to transform these games into games of finite duration without losing too much information. We observe in most games of infinite duration that the relevant knowledge, which determines the payoffs of the players, is not really infinite: it can be just the fact that a state is reached and, in more complicated cases, we just have to know in which strongly connected component of the game graph the game ends (or where the infinite part of the play will take place). To answer the challenge of transforming finite-duration games, we introduce the concept of power matrix which is our main contribution.

In Section 1, we introduce basics about game theory. In Section 2, we illustrate why the transformation of an infinite-duration game into a finite one is not a simple problem. In Section 3, we introduce our main concept: the power matrix, which is an answer to the problem raised in Section 2. Finally we give, in Section 4, some examples of usages of the power matrix and, in Section 5, we justify these examples by demonstrating some properties of the power matrix.

1 About game theory

In this section we give some basic definitions from game theory. For a more detailed introduction on 2-player games, we refer the reader to [7]. Background on extensive games can be found in [5].

1.1 Basic definitions

We first define the notion of an *n-player (graph) game* and then we give some basics about game theory, like the notions of *play*, *strategy*, or *consistence*.

Definition 1.1. *An* n-player (graph) game *is a tuple* $G = (V, (V_i)_{i<n}, v_0, E, (\varphi_i)_{i<n})$ *where* $V = \dot{\bigcup}_{i<n} V_i$ *is a set of vertices and* V_i *is a set of* positions *for player* i*. The position* $v_0 \in V$ *is the starting position. The set of edges* $E \subseteq V \times V$ *is the set of* transitions*. Finally,* φ_i *is a* condition*, typically over reached vertices, that player* i *wants to ensure.*

A *play* is a sequence of positions $\rho = v_0 v_1 v_2 \dots$ such that $(v_k, v_{k+1}) \in E$ for each k. Plays are formed interactively by the players, which we call $0..n$ (in an n-player game). We can have either finite or infinite plays. A *strategy* for player i starting from v_0 is a function $s : V^+ \to V$ which, for each prefix $v_0 \dots v_p$ where $v_p \in V_i$, gives a position r such that $(v_p, r) \in E$. A play $\rho = v_0 v_1 v_2 \dots$ is *consistent* with a strategy s (for player i) if, for each k, such that $v_k \in V_i$, we have $s(v_k) = v_{k+1}$. A strategy s for player i is *positional* (or memoryless) if $s(v_0 \dots v_p)$ depends only of v_p. Such a strategy can be described by a function $f : V_i \to V$. A strategy s for player i such that every play consistent with s satisfies φ_i is called a winning strategy for player i.

1.2 Some fundamental games

Here is a description of some frequently used games, see [7] for other games. To describe these games we describe the condition φ_i that player i wants to ensure. We first introduce guarantee and reachability games, which are games of the same kind in zero-sum and non-zero-sum version respectively.

Definition 1.2. *Let $G = (V, E)$ be a game graph. Let F be subset of V. A* guarantee game *is a 2-player game over G such that player 0 wins a play $\rho = v_0 v_1 ...$ if and only if there is k such that v_k is in F. Player 1 wins if and only if player 0 does not win.*

One can prove that guarantee games are *determined*: for every starting position player 0 or player 1 has a winning strategy. In these games it is possible to compute the *winning regions* (the set of all the starting positions from which a given player has a winning strategy) and the corresponding winning strategies in polynomial time (see [7] for a proof).

Definition 1.3. *Let $G = (V, E)$ be a game graph. Let $(F_i)_{i<n}$ be subsets of V. A* reachability game *is an n-player game over G where player i's goal is to reach at least one vertex from F_i.*

Definition 1.4. *Let $G = (V, E)$ be a game graph. Let $c : V \to \mathbb{N}$ be a priority function. A* parity game *is a 2-player game over G such that player 0 wins if and only if the smallest priority seen infinitely many times is even. Else player 1 wins.*

One can prove that parity games are determined. It is also possible to compute the winning regions and corresponding winning strategies.

1.3 Other definitions

A well-known kind of games are zero-sum game, they are most intuitive.

Definition 1.5. *A* 2-player zero-sum game *is a game such that for each play a payoff is associated to each player (by an* utility function*) and the sum of the payoffs for the two players for a given play is zero. In other words: if one player wins the other loses and each play is won by a player.*

The main concept proposed in this paper is a normal-form game so we give a definition for this way to represent a game.

Definition 1.6. *A* normal-form game *or game in normal form (for two players) consists of a matrix and a function. The rows of the matrix are strategies for player 0 and the columns are strategies for player 1. The entries of the matrix are called* consequences *(or outcomes). The function is called* payoff function *or utility function and associates a real-valued payoff to each consequence.*

It is possible to define normal-form games for more than two players. For n players the matrix is n-dimensional, each dimension corresponding to a player. A normal-form game is also called *strategic game*; a formal definition can be found in [5].

We also have to use imperfect-information games. Intuitively, an imperfect-information game is a game where some positions are indistinguishable for a given player.

Definition 1.7. *We call* imperfect-information game *a game where some positions are indistinguishable for a player. That is, the players can not use the fact that they are on this position to set up their strategy.*

Imperfect-information games are in reality more complicated; you can find more about it (in particular formal definitions) in [8] and [6] for example. In [2] is an example of the usefulness of imperfect information.

In the following, in a game involving n players, we refer to a list of elements $x = (x^i)_{i<n}$, one for each player, as a profile. For any such profile we write x^{-i} to denote the list $(x^j)_{j<n,j\neq i}$ of elements in x for each player except i. Given an element x^i and a list x^{-i}, we denote by (x^i, x^{-i}) the profile $(x^i)_{i<n}$. We also call S^i the set of strategies for player i.

2 Motivations for our study

In computer science, to deal with infinity is always a challenge. In game theory, there are two main sources of infinity: the game graph could be infinite or it could be finite but with potentially infinite plays. We focus on the second aspect which is very frequent: in many games (like parity games) the condition to win is based on infinite plays but also in more simple games it is possible to see infinite plays. For example in guarantee games one of the players wants to reach a set, it seems that the plays will be finite: as soon as the set is reached the game ends. But the other player wants to ensure that is opponent will not reach a set: infinite plays are good for him and, if he can force the game to stay in a cycle, he will do it.

As we saw, infinite plays are frequent. Our goal is to transform games with potentially infinite plays into games with only finite plays without too much loss of information. The main observation, which lets us think that it is possible, is the fact that one does not really need to know what happens in a strongly connected component of the game graph, it is enough to know whether a play will stay or not in the component.

The best we could hope for is to transform our game G into a new one G' and to find a mapping f from the strategies over G to the strategies over G' such that given a strategy s^i over G for player i we have, for each s^{-i}, the following equality: $o(s^i, s^{-i}) = o'(f(s^i), f(s^{-i}))$ (where o is the outcome corresponding to the only play consistent with the strategy profile s). Unfortunately this is impossible, as shown in the following lemma.

Definition 2.1. *Two strategies s^i and r^i for player i are* outcome equivalent *if and only if for every s^{-i} the outcome corresponding to the only play consistent with the profile of strategies (s^i, s^{-i}) has exactly the same outcome as the only play consistent with the profile of strategies (r^i, s^{-i}).*

Lemma 2.1. *There exist games with infinitely many strategies that are not outcome equivalent.*

Proof. In Figure 1 is a game graph. Circles belong to player 0 and squares to player 1. We assume that there are two different possible outcomes, o_1 and o_2, in this game. If a play reaches the position o then the outcome is o_1 and if it is infinite, then the outcome is o_2.

In this game we can construct an infinite set of strategies $S_0 = \{s_1^0, s_2^0, s_3^0...\}$ for player 0 such that each of these strategies is in a different equivalence class for outcome equivalence. Let s_k^0 be the strategy defined in the following way:

- the first time position 1 is reached, go to position 2;
- if position 1 is reached again, go to position o if, and only if, position 2 has been seen less than $k + 1$ times before; else go to position 2.

To prove that each of these strategies is in a different equivalence class, we construct an infinite set of strategies $S_1 = \{s_1^1, s_2^1, s_3^1...\}$ for player 1. Let s_k^1 be the strategy defined in the following way:

- at the $k - 1$ first visits to position 2, go to position 3;
- at the $k - 1$ first visits to position 3, go to position 2;

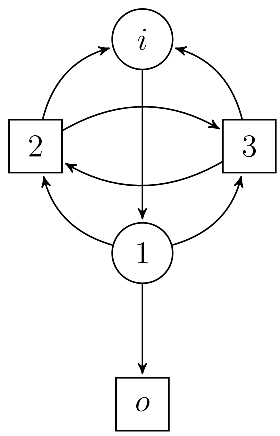

Figure 1: A problematic configuration

- at the k^{th} visit to position 2, go to position i.

If we play with strategies s^0_ℓ and s^1_m two cases are possible: if $\ell \leq m$, then the outcome is o_1 and else, if $l > m$, it is o_2. This proves that all the strategies from S_0 are in different equivalence classes. Hence, the lemma is proved.

□

One can notice that this proof only uses the configuration of the game graph, there is no assumption about the outcomes (except that there are more than one) and the players (except that there are at least two). Actually, this lemma holds even in very simple games, like zero-sum games.

If we want to transform our games, we have to accept a loss of precision about outcomes. We take inspiration in a very intuitive representation of zero-sum games: they are determined so they can be represented as a game where a player chose to play a winning strategy or not. In Figure 2 is a representation of a generic zero sum game in this way. In this representation the player with circle is the one who has a winning strategy, he first chooses whether to use it or not and then the other player chooses his strategy, which finally leads to an outcome.

3 The concept of power matrix

Due to Lemma 2.1, we know that it is impossible to transform a game with potentially infinite plays into one with only finite plays in a way that respects all outcomes. Thus, we need to be more general, instead of looking to outcomes we

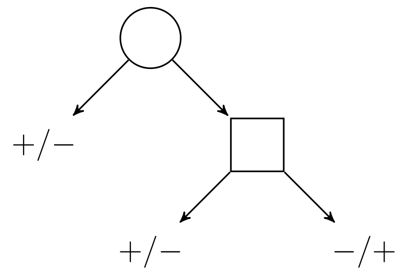

Figure 2: A representation of a typical zero-sum game

choose to consider sets of possible outcomes. For this reason, we introduce the concept of power matrix.

The power matrix is a normal-form game. Therefore, it is an imperfect-information game: the players choose their strategies at the same time, without any knowledge about the other players' choice. We have to describe this game in terms of Definition 1.6. We only define the matrix of this normal-form game, in fact the payoff function can be chosen in different ways, depending of what we want to do with the power matrix.

Let $G = (V, E)$ be a game graph for n players. The power matrix is constructed from G with respect to a finite set $W \subseteq V$ of targets and a starting position $v \in V$. We assume that the game ends as soon as a target is reached and that it ends only if a target is reached. Intuitively, choosing a strategy in the power matrix corresponds to force the plays over G to reach a subset of W.

To describe the power matrix formally, we have to introduce some notions.

Definition 3.1. *Let s be a complete strategy profile. We define the* consequence *of this profile,* $\mathrm{cons}(s) \in W \cup \{\infty\}$*, as the element of the set $W \subseteq V$ of targets reached by the unique play consistent with all the element of the profile s or ∞ if this play is infinite.*

Definition 3.2. *Let s^i be a strategy for player i. We define the* consequence *of this strategy,* $\mathrm{cons}(s^i) \subseteq W \cup \{\infty\}$*, as the set of elements from W reachable by plays consistent with s^i with the addition of ∞ if at least one of these plays is infinite, i.e.,*

$$\mathrm{cons}(s^i) = \{w \in W \cup \{\infty\} \mid \textit{there exists } s^{-i} \textit{such that } \mathrm{cons}((s^i, s^{-i})) = w\}.$$

The notion of consequence of a strategy leads to an equivalence relation between strategies that we use to construct our power matrix. The non-empty equivalence classes of this relation are the strategies over the power matrix.

Definition 3.3. *Let s^i and r^i be two strategies for player i. We say that s^i and r^i are* power equivalent *if and only if* $\mathrm{cons}(s^i) = \mathrm{cons}(r^i)$. *We denote this by* $s^i \sim r^i$.

Definition 3.4. *A* power strategy *is an equivalence class for the power equivalence. Let C be a subset of $W \subseteq V$ a set of targets. We denote by $[C]^i$ the power strategy for player i such that, for $s^i \in [C]^i$, we have* $\mathrm{cons}(s^i) = C$, *i.e.,*

$$[C]^i = \{s^i \in S^i \mid \textit{for all } (s^{-i}), \mathrm{cons}((s^i, s^{-i})) \in C \\ \textit{for all } c \in C, \textit{exists } s^{-i}, \mathrm{cons}((s^i, s^{-i})) = c\ \}.$$

Lemma 3.1. *In every game, there is a finite number of power strategies for each player.*

Proof. Each power strategy is defined by a subset of $W \cup \{\infty\}$. Because W is a finite set, there is a finite number of subsets (at most $2^{|W|+1}$, where $|W|$ is the number of elements in W). □

As we said, the non-empty power strategies are the strategies in the power matrix, according to Definition 1.6. Because the number of these strategies is finite we obtain a useful corollary about the power matrix.

Corollary 3.1. *The power matrix of every game is finite.*

The strategies in the power matrix are now well defined. We just have to describe the entries of this matrix. These entries are the consequences of a profile of power strategies.

Definition 3.5. *Let p be a profile of power strategies, which means that each p^i is a power strategy: $p^i = [C]^i$, and C is not necessarily the same for all the p^i. We call* consequences *of this profile the following set:*

$$\mathrm{cons}(p) = \bigcup \{\mathrm{cons}(s) \mid s \in \bigtimes_{i<n} p^i\}.$$

The set $\mathrm{cons}(p)$ is the set of consequences of all the complete profiles constructable using strategies from the power strategies of p. It is clear that each entry of the power matrix is included in the intersection of the power strategies corresponding to this entry. But these two sets are not always equal, as we show in an example of the Section 4. Formally we have $\mathrm{cons}(p) \subseteq \bigcap_{i<n} p^i$, where p is a profile of power strategies.

In this section we described the concept, of a power matrix abstractly. This matrix is constructed from a game. In the following we explain how it could be used and why it is useful for our study.

4 Applications of the power matrix

In this part we give some examples of using of the power matrix described in Section 3. This show the usefulness of this concept and, in Section 5, we give a property which makes the interest of this concept even stronger.

4.1 Power of a player

The most simple application of the power matrix is to use it as a representation of a whole game, in the following way.

It is clear that each game graph can be associated with a (potentially infinite) tree where a path is a play over the game graph (see [5]). The leaves of this tree are the outcomes associated with each play. Defining W as the set of all the leaves, one can construct a power matrix associated with this tree: the power strategies are the sets of outcomes that players can enforce.

In fact the power strategies of a player represent the influence of this player over the game, that is what we call *power of a player*.

For example, as we saw in Section 2, in a zero-sum game one player has all the power: he can chose to win the game or not.

4.2 Replacing components of a game

An important fact about the power matrix is that it is possible to replace some components of a given game by their power matrix (more precisely the graph representation of their power matrix). We now give the definition of a replaceable components and of the graph representation of a power matrix and explain how to switch between them.

Definition 4.1. *The* component *of the game graph* $G = (V, E)$ *with input* $I \subseteq V$ *and output* $O \subseteq V$ *the sub-graph of* G *which contains all the paths from* I *to* O *and only these paths. A component is a game graph. A* replaceable component *is a component* $G' = (V', E')$ *such that there is no edge between* $V' \setminus (I \cup O)$ *and* $V \setminus V'$*.*

In Figure 3 is a game graph for a 2-player game (positions for player 0 are circles and positions for player 1 are squares). The component with $I = \{i\}$ and $O = \{a, b\}$ is a replaceable component.

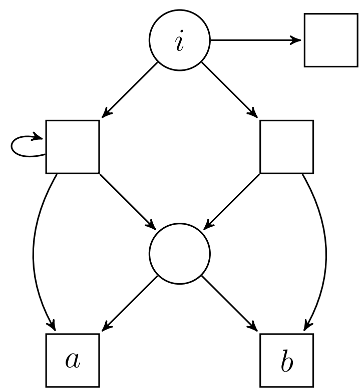

Figure 3: Game graph and replaceable component

It is also possible to represent a power matrix as a game with imperfect information. The imperfect information reflects the fact that the players choose their strategies at the same time.

Definition 4.2. *A game graph of a power matrix or* power graph *is a tree. The root belongs to the first player. There is exactly one level of the tree for each player. From each position which belongs to him a player can choose between all his power strategies. All the positions of a given player are indistinguishable for him. Each leaf of this tree correspond to an entry of the power matrix, associated to the power strategies chosen by the players to reach this leaf.*

The power matrix associated with the replaceable component of Figure 3 is represented in Table 1 (player 0 controls the rows and player 1 the columns).

A corresponding power graph (with root for player 0) is represented in Figure 4. In this game player 0 chooses first a power strategy, which leads to one of the positions for player 1. Then player one chooses a power strategy, which leads

	$\{a,b\}$	$\{\infty,b\}$	$\{\infty,a,b\}$
$\{a,\infty\}$	$\{a\}$	$\{\infty\}$	$\{\infty\}$
$\{a,\infty,b\}$	$\{a,b\}$	$\{\infty\}$	$\{\infty\}$
$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$
$\{a,b\}$	$\{a,b\}$	$\{b\}$	$\{a\}$

Table 1: Power matrix

to one of the leaves. All the positions for player 1 are equivalent (represented by dashed lines). For example, if player 0 chooses the power strategy $\{b\}$, then whatever the player 1 chooses, the leaf reached will be $\{b\}$, which is represented at the right of Figure 4.

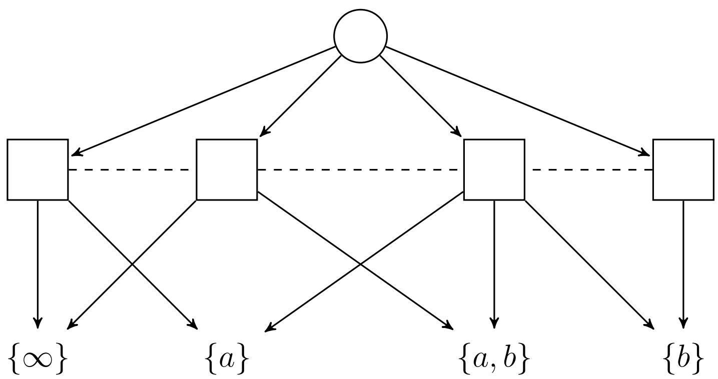

Figure 4: Power graph

We introduced the notion of a replaceable component of a game graph and game graphs of power matrices, called power graphs. We now show how to replace such a component by a corresponding power graph.

First, it is necessary to construct a power graph $\hat{G}$ of the power matrix corresponding to the component. Then, one just needs to erase the edges and vertices of the component, except the input and output vertices, add an edge from each input to the root of $\hat{G}$ and join the leaves of $\hat{G}$ to the outputs. As we explained in Definition 4.2 each of these leaves represents a subset of the outputs. Every leaf will have an edge to each output from the subset it represents (if one output is ∞ we just add a new position with none outgoing edge to represent this possibility). A new and neutral player, that is a player with no goal, will control the leaves. This is a form of non-determinism: none of the initial players can chose the final output.

In Figure 5 is represented the graph of Figure 3 where the component studied before is replaced with a power graph corresponding to the power matrix of Figure 4. The positions which are neither circles nor squares belong to a third player which has no objective in the game: it represents non-determinism.

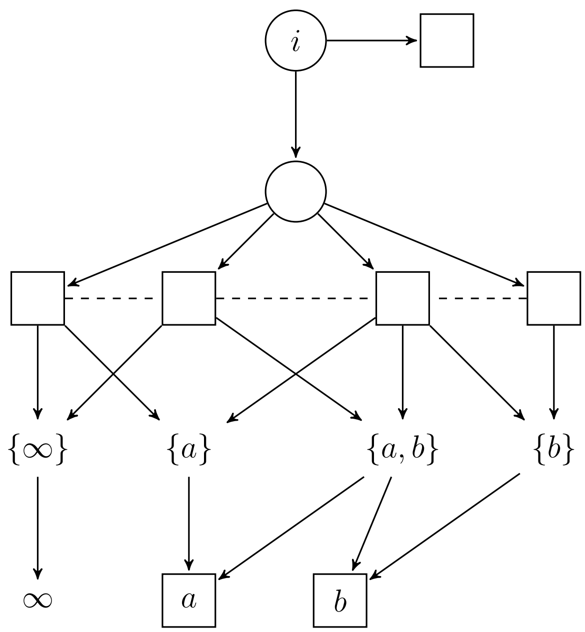

Figure 5: The new game graph

4.3 Application to guarantee games

The replacement of components by power graphs is a very powerful tool which allows us to transform certain games with potentially infinite plays into games with only finite plays. We call this transformation *disconnection* of a game. We give an example with guarantee games (see Definition 1.2).

The idea is to replace each strongly connected component of the game graph by a corresponding power graph.

For the following we fix a game graph $G = (V, E)$ and a subset F of V. We focus on a guarantee game over G with F used as the F of Definition 1.2.

Definition 4.3. *A* strongly connected component *of G is a component $G' =$*

(V', E') *such that for each pair* (u, v) *of vertices from* V' *there is a path from* u *to* v *in* G' *and all the paths from* u *to* v *are in* G'.

We also have to define the sets I and O for a given strongly connected component. Let $G' = (V', E')$ be a strongly connected opponent of G. We define I as the set of all the vertices i of V' for which there is a vertex $v \in V \setminus V'$ such that (v, i) is in E. Likewise, we define O as the set of all the vertices o of V' for which there is a vertex v of $V \setminus V'$ such that (o, v) is in E, plus the elements of $V' \cap F$.

Once we have defined I and O for a given strongly connected component, the transformation of a guarantee game with potentially infinite plays into a guarantee game with only finite plays is very simple: one has only to replace the strongly connected components one by one. When no strongly connected component remains, the new game graph is a directed acyclic graph. Hence, all the plays over this graph are finite. Therefore this transformation keeps all the information needed to solve the game: the vertices of F and the paths to reach them.

In the next part we show that the dominance relation between strategies is preserved when we replace a component. In particular, if there is a strategy which ensures player i to win over G, then there is a strategy which ensures him to win over the new game. It validates the transformation.

5 An interesting property of the power matrix

In this section we explain the dominance relation between strategies and we show that the power matrix preserves this relation.

5.1 Dominance between strategies

The notion of dominance between strategies is derived from an order over sets of outcomes.

Definition 5.1. *Given a game* G *and an order* $<$ *over the sets of possible outcomes we say that a set* X *of outcomes* dominates *a set* Y *of outcomes if and only if* $Y < X$. *In this case we also say that* Y *is dominated by* X.

We can extend this notion of dominance between sets of outcomes to a notion of dominance between strategies.

Definition 5.2. *In a game G, we say that a strategy s^i, for player i, dominates a strategy r^i if and only if the set X of outcomes associated with all the plays consistent with s^i and the set of outcomes Y associated with all the plays consistent with r^i are such that X dominates Y.*

This is a very intuitive notion: if a player can order the sets of possible outcomes (which means that he has preferences between them) then he can also order his strategies and determine which strategy or set of strategies he prefers to use.

5.2 The basic property

We first give a basic property about the transformation proposed in Section 4.1.

Lemma 5.1. *The dominance relation between strategies is preserved by the transformation of Section 4.1.*

This means that:

1. if a strategy s^i for player i dominates a strategy r^i in a game G, then the power strategies S^i associated with s^i and R^i associated with r^i are such that S^i dominates R^i in the new game;

2. if a power strategy S^i dominates a power strategy R^i in the new game then all the strategies corresponding to S^i dominate the strategies corresponding to R^i in G.

Proof. The power strategies are exactly the sets of outcomes of Definition 5.2. □

In particular the lemma implies that, if there is a winning strategy in a game G, then there is a winning strategy in the corresponding power matrix (as defined in Section 4.1).

5.3 Recursive aspect of the property

We prove in this section that, in guarantee games, the replacement of a component by a corresponding power graph, as suggested in Sections 4.2 and 4.3 preserves the dominance between strategies.

Lemma 5.2. *The dominance relation between strategies is preserved by the disconnection of a game.*

Let G be a guarantee game. Let $\hat{G}$ be the new game constructed by replacing a strongly connected component in the game graph of G as described in Section 4.3. Formally we want to prove that:

1. if a strategy s^i for player i dominates a strategy r^i for player i in G then the strategy $\hat{s}^i$ corresponding to s^i dominates the strategy $\hat{r}^i$ corresponding to r^i in $\hat{G}$;

2. if in $\hat{G}$ a strategy $\hat{s}^i$ for player i dominates a strategy $\hat{r}^i$ for player i then in G all the strategies s^i corresponding to $\hat{s}^i$ dominate the strategies r^i corresponding to $\hat{r}^i$.

Proof. It is sufficient to notice that a given vertex is reachable using a given strategy in G if, and only if, it is reachable using the corresponding power strategy in $\hat{G}$. □

The new game, constructed from a guarantee game by replacing a strongly connected component by a corresponding power graph, is also a guarantee game. Hence if we replace all the strongly connected components we can ensure that the dominance relation between strategies is preserved from the original game (by applying Lemma 5.2 after each atomic transformation). This validates the use of the power matrix, in the case of guarantee games.

Conclusion

In this paper we have shown that, without making assumptions about the way the players play, there is in general an infinite number of equivalence classes of strategies for outcome equivalence, even in zero-sum games (Lemma 2.1). According to this, it is not possible in general to transform a game into a new one with a finite number of strategies for each player such that there is a mapping for strategies from the first game to the second one which respects outcomes.

The main contribution of this paper is the concept of power matrix. It is a response to the problem opened by Lemma 2.1. We introduced this concept to transform a game with potentially infinite plays into a new one with only finite plays, such that dominance between strategies is preserved from one game to another. This power matrix is very general and, in particular, it does not require assumptions about the way the players will play. Because this concept is rather abstract, we also gave some applications. Associated with the preservation of dominance between strategies these examples show the usefulness of our concept.

Here, we only used the power matrix to transform guarantee game which are zero-sum games. Hence, we did not really demonstrate the interest of power matrix, which is the fact that it is independent of the players' goals. One future project is to use our power matrix to transform non-zero-sum games first reachability games which are close to guarantee games and then other kind of games like, for example, parity games. It could also be interesting to see if, in the transformation of Section 4.2, it is possible to make the new game determinist.

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