A short introduction to ATL-like logics with resources

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Logics for resource-bounded agents

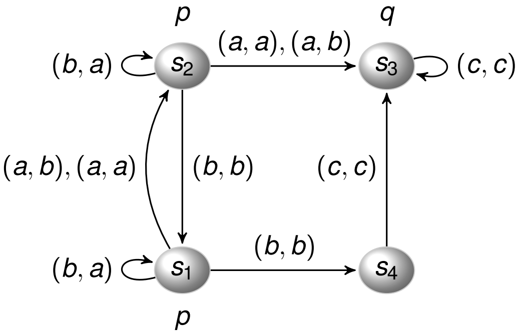
- ATL-like logics with models where transitions have costs/rewards and resource requirements are expressed in the syntax.

- Model-checking problems for such logics are often undecidable as games on VASS are often undecidable.

- Many existing resource logics:
  - RBTL* [Bulling & Farwer, CLIMA X '09]
  - QATL* [Bulling & Goranko, EPTCS 2013]
  - RB$\pm$ATL [Alechina et al., ECAI'14]
  - etc.

- Other logics for resource-bounded agents: step logic, justification logic, etc.

Concurrent game structures



$Agt = \{1, 2\}$

$S = \{s_1, s_2, s_3, s_4\}$

$Act = \{a, b, c\}$

- Action manager $\mathtt{act} : Agt \times S \to \mathcal{P}(Act) \setminus \{\emptyset\}$.
  $\mathtt{act}(1, s_3) = \{c\}$.
- Transition function $\delta : S \times (Agt \to Act) \to S$.
  $\delta(s_4, [1 \mapsto c, 2 \mapsto c]) = s_3$.
- Labelling $L : S \to \mathcal{P}(\mathrm{PROP})$.

Basic concepts: joint actions and computations

- $\mathfrak{f} : A \to Act$: joint action by $A \subseteq Agt$ in s.
  Proviso: for all $a \in A$, we have $\mathfrak{f}(a) \in \mathtt{act}(a, s)$.

- $D_A(s)$: set of joint actions by A in s.

  $$\mathtt{out}(s, \mathfrak{f}) \stackrel{\text{def}}{=} \{s' \in S \mid \exists\, \mathfrak{g} \in D_{Agt}(s) \text{ s.t. } \mathfrak{f} \sqsubseteq \mathfrak{g} \;\&\; s' = \delta(s, \mathfrak{g})\}$$

- Computation $\lambda = s_0 \xrightarrow{\mathfrak{f}_0} s_1 \xrightarrow{\mathfrak{f}_1} s_2 \ldots$ such that for all i, we have $s_{i+1} \in \delta(s_i, \mathfrak{f}_i)$.

- Linear model $L(s_0) \to L(s_1) \to L(s_2) \cdots$.

Basic concepts: strategies

- A strategy F_A for A is a map from the set of finite computations to the set of joint actions by A such that $F_A(s_0 \xrightarrow{\mathfrak{f}_0} s_1 \cdots \xrightarrow{\mathfrak{f}_{n-1}} s_n) \in D_A(s_n)$.

- $\lambda = s_0 \xrightarrow{\mathfrak{f}_0} s_1 \xrightarrow{\mathfrak{f}_1} s_2 \cdots$ respects $F_A \overset{\text{def}}{\Leftrightarrow} \forall\, i < |\lambda|$,

$$s_{i+1} \in \mathtt{out}(s_i, F_A(s_0 \xrightarrow{\mathfrak{f}_0} s_1 \ldots \xrightarrow{\mathfrak{f}_{i-1}} s_i))$$

- λ respecting F_A is maximal whenever λ cannot be extended further while respecting the strategy.

- $\mathtt{comp}(s, F_A)$: max. computations from s respecting F_A.

The logic ATL

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle A \rangle\rangle \, \mathsf{X}\phi \mid \langle\langle A \rangle\rangle \, \mathsf{G}\phi \mid \langle\langle A \rangle\rangle \, \phi \mathsf{U} \phi$$

$$p \in \mathrm{PROP} \quad A \subseteq Agt$$

$\mathfrak{M}, s \models p \quad \stackrel{\text{def}}{\Leftrightarrow} \quad s \in L(p)$

$\mathfrak{M}, s \models \langle\langle A \rangle\rangle \mathsf{X}\phi \quad \stackrel{\text{def}}{\Leftrightarrow}$ there is a strategy F_A s.t.
for all $s_0 \xrightarrow{\mathfrak{f}_0} s_1 \ldots \in \mathtt{comp}(s, F_A)$,
we have $\mathfrak{M}, s_1 \models \phi$

$\mathfrak{M}, s \models \langle\langle A \rangle\rangle \phi_1 \mathsf{U} \phi_2 \quad \stackrel{\text{def}}{\Leftrightarrow}$ there is a strategy F_A s.t. for all
$\lambda = s_0 \xrightarrow{\mathfrak{f}_0} s_1 \ldots \in \mathtt{comp}(s, F_A)$,
there is some $i < |\lambda|$ s.t. $\mathfrak{M}, s_i \models \phi_2$ and
for all $j \in [0, i-1]$, we have $\mathfrak{M}, s_j \models \phi_1$.

Model-checking problem

- Model-checking problem for ATL:

  Input: ϕ in ATL, a finite CGS $\mathfrak{M}$ and a state s,

  Question: $\mathfrak{M}, s \models \phi$?

- Model-checking problem for ATL is P-complete.
  Labeling algorithm. [Alur & Henzinger & Kupferman, JACM 2002]

- ATL* = ATL + all path formulae à la CTL*.

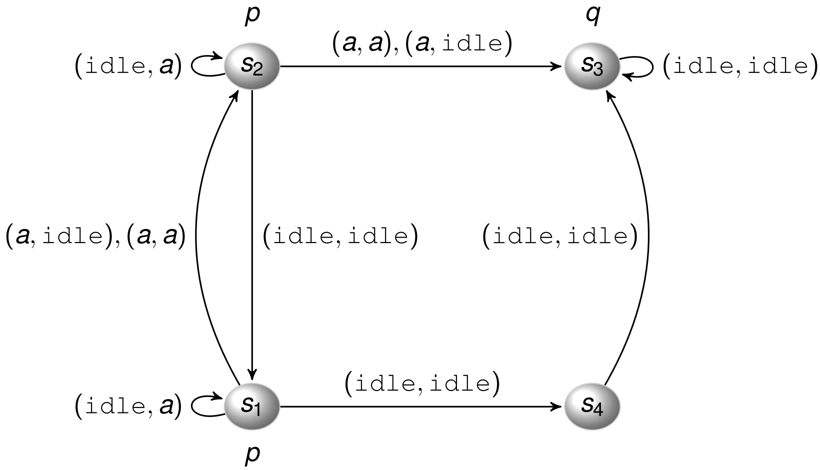
- Model-checking problem for ATL* is 2EXPTIME-complete.

Resource-bounded concurrent game structures

$\boxed{\text{Concurrent game structures + resources (counters)}}$

- Number r of resources/counters.
- Partial cost function $\texttt{cost} : S \times Agt \times Act \rightarrow \mathbb{Z}^r$.
- Action $\texttt{idle} \in \texttt{act}(a, s)$ with no cost.
- Given a joint action $\mathfrak{f} : A \rightarrow Act$,

$$\texttt{cost}_A(s, \mathfrak{f}) \stackrel{\text{def}}{=} \sum_{a \in A} \texttt{cost}(s, a, \mathfrak{f}(a))$$



$\texttt{cost}(s_2, 1, a) = (1, 1, 1, 1)$ $\texttt{cost}(s_2, 2, a) = (-2, 1, -3, 1)$

$$\texttt{cost}_{\{1,2\}}(s_2, [1 \mapsto a, 2 \mapsto a]) = (-1, 2, -2, 2)$$

b-strategies

- Initial budget $\mathbf{b} \in (\mathbb{N} \cup \{\omega\})^r$.
- $\lambda = s_0 \xrightarrow{\mathfrak{f}_0} s_1 \xrightarrow{\mathfrak{f}_1} s_2 \ldots$ in $\mathtt{comp}(s, F_A)$ is **b**-consistent:
  - $\mathbf{v}_0 \stackrel{\text{def}}{=} \mathbf{b}$,
  - $\mathbf{v}_{i+1} \stackrel{\text{def}}{=} \mathbf{v}_i + \mathtt{cost}_A(s_i, F_A(s_0 \xrightarrow{\mathfrak{f}_0} s_1 \ldots \xrightarrow{\mathfrak{f}_{i-1}} s_i))$,
  - for all i, $\mathbf{0} \preceq \mathbf{v}_i$.

$\boxed{\text{Asymmetry between } A \text{ and } (Agt \setminus A)}$

- $\mathtt{comp}(s, F_A, \mathbf{b})$: set of all the **b**-consistent computations.
- F_A is a **b**-strategy w.r.t. s $\stackrel{\text{def}}{\Leftrightarrow}$

$$\mathtt{comp}(s, F_A) = \mathtt{comp}(s, F_A, \mathbf{b})$$

The logic RB±ATL (Agt, r) [Alechina et al., ECAI'14]

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle A^{\mathbf{b}} \rangle\rangle\, \mathsf{X}\phi \mid \langle\langle A^{\mathbf{b}} \rangle\rangle\, \mathsf{G}\phi \mid \langle\langle A^{\mathbf{b}} \rangle\rangle\, \phi \mathsf{U} \phi$$

$$p \in \mathrm{PROP} \quad A \subseteq Agt \quad \mathbf{b} \in (\mathbb{N} \cup \{\omega\})^r$$

$\mathfrak{M}, s \models p \quad \overset{\text{def}}{\Leftrightarrow} \quad s \in L(p)$

$\mathfrak{M}, s \models \langle\langle A^{\mathbf{b}} \rangle\rangle \mathsf{X}\phi \quad \overset{\text{def}}{\Leftrightarrow}$ there is a $\boxed{\mathbf{b}\text{-strategy}}$ F_A w.r.t. s s.t. for all $s_0 \xrightarrow{\mathfrak{f}_0} s_1 \ldots \in \mathtt{comp}(s, F_A)$, we have $\mathfrak{M}, s_1 \models \phi$

$\mathfrak{M}, s \models \langle\langle A^{\mathbf{b}} \rangle\rangle \phi_1 \mathsf{U} \phi_2 \quad \overset{\text{def}}{\Leftrightarrow}$ there is a $\boxed{\mathbf{b}\text{-strategy}}$ F_A w.r.t. s s.t. for all $\lambda = s_0 \xrightarrow{\mathfrak{f}_0} s_1 \ldots \in \mathtt{comp}(s, F_A)$ there is some $i < |\lambda|$ s.t. $\mathfrak{M}, s_i \models \phi_2$ and for all $j \in [0, i-1]$, we have $\mathfrak{M}, s_j \models \phi_1$.

Alternative semantics

- In RB±ATL, $\texttt{comp}(s, F_A) = \texttt{comp}(s, F_A, \mathbf{b})$ implies the maximal computations are infinite.
- Infinite semantics: arbitrary strategy but quantifications over infinite computations only.
- Finite semantics: arbitrary strategy but quantifications over maximal computations only.

Resource-bounded reasoners for AI

- RB$\pm$ATL is one of the logics for reasoning about resources. See papers in AAAI, IJCAI, ECAI, etc.
- Relationships with counter machines known for establishing undecidability or complexity lower bounds.
- Various flavours of resource-bounded logics exist: RBCL, RAL, PRB-ATL, etc.

Alternating VASS [Courtois & Schmitz, MFCS'14]

- Alternating VASS $\mathcal{A} = (Q, r, R_1, R_2)$:
  - R_1 is a finite subset of $Q \times \mathbb{Z}^r \times Q$. (unary rules)
  - R_2 is a finite subset of $\bigcup_{\beta \geq 2} Q^\beta$ (fork rules)
- Proof: tree labelled by elements in $Q \times \mathbb{N}^r$ following the rules in $\mathcal{A}$.

$$\dfrac{\dfrac{\dfrac{\vdots}{(q_3,(4,8))}}{(q_2,(1,5))} \qquad \dfrac{\dfrac{\vdots}{(q_0,(0,8))}}{(q_1,(1,5))}}{\dfrac{(q_0,(1,5))}{(q_1,(2,2))}}$$

$$q_1 \xrightarrow{(-1,+3)} q_0 \qquad q_0 \to q_1, q_2 \qquad q_2 \xrightarrow{(+3,+3)} q_3$$

Decision problems

- State reachability problem for AVASS:

  Input: AVASS $\mathcal{A}$, control states q_0 and q_f,

  Question: is there a finite proof of AVASS with root $(q_0, \mathbf{0})$ and each leaf belongs to $\{q_f\} \times \mathbb{N}^r$?

- Non-termination problem for AVASS:

  Input: $\mathcal{A}$, q_0,

  Question: is there a proof with root $(q_0, \mathbf{0})$ and all the maximal branches are infinite?

VASS games with asymmetry between the two players

Main Correspondences

RB±ATL	Alternating VASS
Logic in AI	Verification games
proponent restriction condition	updates in R_1 / no update in R_2
computation tree for F_A	proof
formulae in the scope of $\langle\langle A^{\mathbf{b}} \rangle\rangle$	monotone objectives

- From RB±ATL model-checking to the state reachability and the non-termination problems for AVASS.
- From RB±ATL* model-checking to the parity games for AVASS.
- Parameters synthesis thanks to the computation of the Pareto frontier of parity games.

See [Abdulla et al., CONCUR'13]

Complexity of RB±ATL fragments

$r \backslash \mathrm{card}(Agt)$	arbitrary	2	1
arbitrary	**2EXPTIME-c.**	**2EXPTIME-c.**	**EXPSPACE-c.**
≥ 4	**EXPTIME-c.**	**EXPTIME-c.**	**PSPACE-c.**
$2, 3$	PSPACE-h. in EXPTIME	PSPACE-h. in EXPTIME	**PSPACE-c.**
1	in PSPACE	in PSPACE	**PTIME-c.**

Complexity characterisations established in

[Alechina et al., JCSS 2017; Alechina et al., RP'16; etc.]

based on the relationships with (A)VASS and results from

[Habermehl, ICATPN'97; Courtois & Schmitz, MFCS'14; Colcombet et al., LICS'17]

Parameterized RB$\pm$ATL*: ParRB$\pm$ATL*

- $\mathbf{b} \in (\mathbb{N} \cup \{\omega\})^r$ replaced by tuples of variables.

$$\langle\langle\{1\}^{(\mathrm{x}_1,\mathrm{x}_2)}\rangle\rangle\top \mathsf{U} q_f \wedge \langle\langle\{2\}^{(\mathrm{x}_2,\mathrm{x}_3)}\rangle\rangle\top \mathsf{U} q'_f$$

- MC problem for ParRB$\pm$ATL*: compute the maps $\mathfrak{v} : \{\mathrm{x}_1, \ldots, \mathrm{x}_n\} \to (\mathbb{N} \cup \{\omega\})$ such that $\mathfrak{M}, s \models \mathfrak{v}(\phi)$.
- Symbolic representation for such maps are computable.

Other temporal logics for AI

- TIME: International Symposium on Temporal Representation and Reasoning
  - Artificial Intelligence
  - Temporal Databases
  - Logic
- Interval temporal logics, ATL-like logics, temporal logics over concrete domains, etc.

Concluding remarks

- Formal relationships between resource-bounded logics and games on alternating VASS.
- Open problems:
  - Parameter synthesis.
  - Complexity for small fragments by bounding further the syntactic resources.
  - Alternative semantics for applications.