

On The Complexity of Resource-Bounded Logics

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Overview

Alternating-Time Temporal Logic

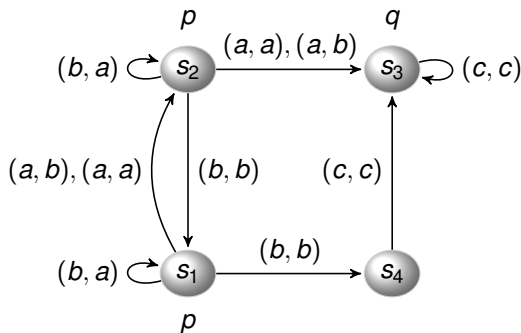
Resource-Bounded Logics

Alternating VASS

Correspondences & Computational Complexity

Alternating-Time Temporal Logic ATL

Concurrent game structures



- ▶ $Ag_t = \{1, 2\}$, $S = \{s_1, s_2, s_3, s_4\}$, $Act = \{a, b, c\}$.
- ▶ Action manager $act : Ag_t \times S \rightarrow \mathcal{P}(Act) \setminus \{\emptyset\}$.
 $act(1, s_3) = \{c\}$.
- ▶ Transition function $\delta : S \times (Ag_t \rightarrow Act) \rightarrow S$.
 $\delta(s_4, [1 \mapsto c, 2 \mapsto c]) = s_3$.
 Undefined $\delta(s_4, [1 \mapsto c, 2 \mapsto a])$.

Basic concepts: joint actions and computations

- ▶ $f : A \rightarrow Act$: joint action by A in s .
Proviso: for all $a \in A$, we have $f(a) \in act(a, s)$.
- ▶ $D_A(s)$: set of joint actions by A in s .

$$out(s, f) \stackrel{\text{def}}{=} \{s' \in S \mid \exists g \in D_{Agt}(s) \text{ s.t. } f \sqsubseteq g \ \& \ s' = \delta(s, g)\}$$

- ▶ Computation $\lambda = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} s_2 \dots$ such that for all i , we have $s_{i+1} \in \delta(s_i, f_i)$.
- ▶ Linear model $L(s_0) \rightarrow L(s_1) \rightarrow L(s_2) \dots$.

Basic concepts: strategies

- ▶ A strategy F_A for A is a map from the set of finite computations to the set of joint actions by A such that

$$F_A(s_0 \xrightarrow{f_0} s_1 \cdots \xrightarrow{f_{n-1}} s_n) \in D_A(s_n).$$

- ▶ $\lambda = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} s_2 \cdots$ respects $F_A \stackrel{\text{def}}{\iff} \forall i < |\lambda|,$

$$s_{i+1} \in \text{out}(s_i, F_A(s_0 \xrightarrow{f_0} s_1 \cdots \xrightarrow{f_{i-1}} s_i))$$

- ▶ λ respecting F_A is maximal whenever λ cannot be extended further while respecting the strategy.
- ▶ $\text{out}(s, F_A)$: set of max. computations from s resp. F_A .

The logic ATL

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle A \rangle\rangle \bigcirc \phi \mid \langle\langle A \rangle\rangle \Box \phi \mid \langle\langle A \rangle\rangle \phi \mathcal{U} \phi$$

$$p \in \text{PROP} \quad A \subseteq \text{Agt}$$

$$\mathfrak{M}, s \models p \quad \stackrel{\text{def}}{\iff} \quad s \in L(p)$$

$$\mathfrak{M}, s \models \langle\langle A \rangle\rangle \bigcirc \phi \quad \stackrel{\text{def}}{\iff} \quad \begin{array}{l} \text{there is a strategy } F_A \text{ s.t.} \\ \text{for all } s_0 \xrightarrow{f_0} s_1 \dots \in \text{out}(s, F_A), \\ \text{we have } \mathfrak{M}, s_1 \models \phi \end{array}$$

$$\mathfrak{M}, s \models \langle\langle A \rangle\rangle \phi_1 \mathcal{U} \phi_2 \quad \stackrel{\text{def}}{\iff} \quad \begin{array}{l} \text{there is a strategy } F_A \text{ s.t. for all} \\ \lambda = s_0 \xrightarrow{f_0} s_1 \dots \in \text{out}(s, F_A), \\ \text{there is some } i < |\lambda| \text{ s.t. } \mathfrak{M}, s_i \models \phi_2 \text{ and} \\ \text{for all } j \in [0, i-1], \text{ we have } \mathfrak{M}, s_j \models \phi_1. \end{array}$$

Model-checking problem

- ▶ CTL = fragment of ATL.
E.g., $\langle\langle \text{Agt} \rangle\rangle \Box p \equiv \text{EG}p$, $\langle\langle \emptyset \rangle\rangle \Box p \equiv \text{AG}p$, ...
- ▶ Model-checking problem for ATL:
 Input: ϕ in ATL, a finite CGS $\mathfrak{M}$ and a state s ,
 Question: $\mathfrak{M}, s \models \phi$?
- ▶ Model-checking problem for ATL is P-complete.
 Labeling algorithm. [Alur & Henzinger & Kupferman, JACM 2002]
- ▶ $\text{ATL}^* = \text{ATL} + \text{all path formulae à la CTL}^*$.
- ▶ Model-checking problem for ATL^* is 2EXPTIME-complete.

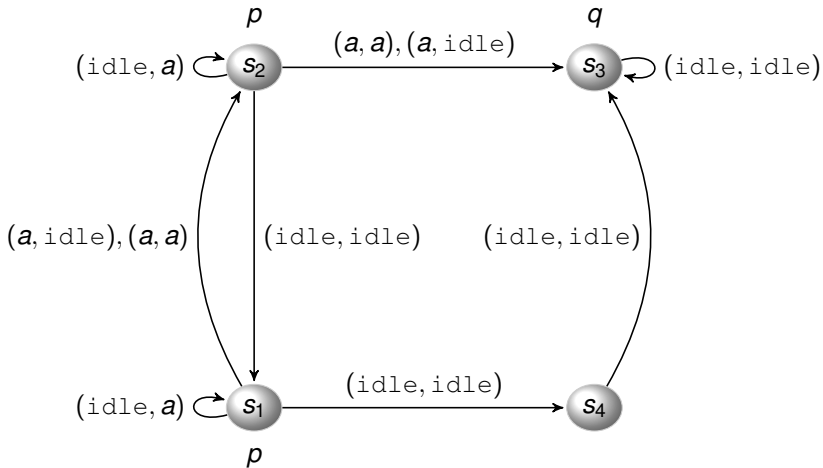
Resource-Bounded Logics

Resource-bounded concurrent game structures

Concurrent game structures + resources (counters)

- ▶ Number r of resources/counters.
- ▶ Partial cost function $\text{cost} : S \times \text{Agt} \times \text{Act} \rightarrow \mathbb{Z}^r$.
- ▶ Action $\text{idle} \in \text{act}(a, s)$ with no cost.
- ▶ Given a joint action $f : A \rightarrow \text{Act}$,

$$\text{cost}_A(s, f) \stackrel{\text{def}}{=} \sum_{a \in A} \text{cost}(s, a, f(a))$$



$$\text{cost}(s_2, 1, a) = (1, 1, 1, 1) \qquad \text{cost}(s_2, 2, a) = (-2, 1, -3, 1)$$

$$\text{cost}_{\{1,2\}}(s_2, [1 \mapsto a, 2 \mapsto a]) = (-1, 2, -2, 2)$$

b-strategies

- ▶ Initial budget $\mathbf{b} \in (\mathbb{N} \cup \{\omega\})^r$.
- ▶ $\lambda = s_0 \xrightarrow{f_0} s_1 \xrightarrow{f_1} s_2 \dots$ in $\text{out}(s, F_A)$ is **b**-consistent:
 - ▶ $\mathbf{v}_0 \stackrel{\text{def}}{=} \mathbf{b}$,
 - ▶ $\mathbf{v}_{i+1} \stackrel{\text{def}}{=} \mathbf{v}_i + \text{cost}_A(s_i, F_A(s_0 \xrightarrow{f_0} s_1 \dots \xrightarrow{f_{i-1}} s_i))$,
 - ▶ for all i , $\mathbf{0} \preceq \mathbf{v}_i$.

Asymmetry between A and $(\text{Agt} \setminus A)$

- ▶ $\text{out}(s, F_A, \mathbf{b})$: set of all the **b**-consistent computations.
- ▶ F_A is a **b**-strategy w.r.t. $s \stackrel{\text{def}}{\iff}$
$$\text{out}(s, F_A) = \text{out}(s, F_A, \mathbf{b})$$
- ▶ $\mathbf{b} \preceq \mathbf{b}'$ and F_A is a **b**-strategy $\rightarrow F_A$ is a **b'**-strategy.

The logic $\text{RB}\pm\text{ATL}$ (Agt, r)

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \langle\langle A^{\mathbf{b}} \rangle\rangle \bigcirc \phi \mid \langle\langle A^{\mathbf{b}} \rangle\rangle \Box \phi \mid \langle\langle A^{\mathbf{b}} \rangle\rangle \phi \mathcal{U} \phi$$

$$p \in \text{PROP} \quad A \subseteq \text{Agt} \quad \mathbf{b} \in (\mathbb{N} \cup \{\omega\})^r$$

$$\mathfrak{M}, s \models p \quad \stackrel{\text{def}}{\Leftrightarrow} \quad s \in L(p)$$

$$\mathfrak{M}, s \models \langle\langle A^{\mathbf{b}} \rangle\rangle \bigcirc \phi \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \text{there is a } \boxed{\mathbf{b}\text{-strategy}} \ F_A \text{ w.r.t. } s$$

s.t. for all $s_0 \xrightarrow{f_0} s_1 \dots \in \text{out}(s, F_A)$,
we have $\mathfrak{M}, s_1 \models \phi$

$$\mathfrak{M}, s \models \langle\langle A^{\mathbf{b}} \rangle\rangle \phi_1 \mathcal{U} \phi_2 \quad \stackrel{\text{def}}{\Leftrightarrow} \quad \text{there is a } \boxed{\mathbf{b}\text{-strategy}} \ F_A \text{ w.r.t. } s$$

s.t. for all $\lambda = s_0 \xrightarrow{f_0} s_1 \dots \in \text{out}(s, F_A)$,
there is some $i < |\lambda|$ s.t.
 $\mathfrak{M}, s_i \models \phi_2$ and for all $j \in [0, i-1]$,
we have $\mathfrak{M}, s_j \models \phi_1$.

Resource-bounded reasoners for AI

- ▶ $RB\pm ATL$ is one of the logics for reasoning about resources. See papers in AAI, IJCAI, ECAI, etc.
- ▶ Relationships with counter machines known for establishing undecidability or complexity lower bounds.
- ▶ Various flavours of resource-bounded logics exist: RBCL, RAL, PRB-ATL, etc.

Model-checking problem

- ▶ The model-checking problem for $\text{RB}\pm\text{ATL}$:

Input: $k, r \geq 1$ (in unary), a formula ϕ in $\text{RB}\pm\text{ATL}([1, k], r)$, a finite $\text{RB}\pm\text{ATL}([1, k], r)$ model $\mathfrak{M}$ and a state s ,

Question: $\mathfrak{M}, s \models \phi$?

- ▶ The model-checking problem for $\text{RB}\pm\text{ATL}$ is decidable.

[Alechina et al., ECAI'14]

- ▶ The question of the computational complexity left open.
EXPSPACE-hardness known from [Alechina et al., TR 2015]
- ▶ Decidability of the problem for $\text{RB}\pm\text{ATL}^*$ left open.

Alternating VASS

Alternating VASS [Courtois & Schmitz, MFCS'14]

- ▶ Alternating VASS $\mathcal{A} = (Q, r, R_1, R_2)$:
 - ▶ R_1 is a finite subset of $Q \times \mathbb{Z}^r \times Q$. (unary rules)
 - ▶ R_2 is a finite subset of $\bigcup_{\beta \geq 2} Q^\beta$ (fork rules)
- ▶ Proof: tree labelled by elements in $Q \times \mathbb{N}^r$ following the rules in $\mathcal{A}$.

$$\begin{array}{c}
 \vdots \qquad \qquad \vdots \\
 \frac{(q_3, (4, 8))}{(q_2, (1, 5))} \quad \frac{(q_0, (0, 8))}{(q_1, (1, 5))} \\
 \hline
 (q_0, (1, 5)) \\
 \hline
 (q_1, (2, 2))
 \end{array}$$

$$q_1 \xrightarrow{(-1, +3)} q_0 \quad q_0 \rightarrow q_1, q_2 \quad q_2 \xrightarrow{(+3, +3)} q_3$$

Decision problems

- ▶ State reachability problem for AVASS:

Input: AVASS $\mathcal{A}$, control states q_0 and q_f ,

Question: is there a finite proof of AVASS with root $(q_0, \mathbf{0})$ and each leaf belongs to $\{q_f\} \times \mathbb{N}^r$?

- ▶ Non-termination problem for AVASS:

Input: $\mathcal{A}$, q_0 ,

Question: is there a proof with root $(q_0, \mathbf{0})$ and all the maximal branches are infinite?

VASS games with asymmetry between the two players

2EXPTIME-completeness

- ▶ Decidability established for asymmetric VASS games in
[Raskin & Samuelides & Begin, ENTCS 2005].
- ▶ 2EXPTIME-hardness for both problems.
[Courtois & Schmitz, MFCS'14]
- ▶ State reachability problem in 2EXPTIME.
[Courtois & Schmitz, MFCS'14]
- ▶ Non-termination problem in 2EXPTIME.
[Jurdziński & Lazić & Schmitz, ICALP'15]

Variants

- ▶ Problems still in 2EXPTIME when:
 - ▶ Reachability is related to a subset $Q_f \subseteq Q$.
 - ▶ The initial configuration is $(q_0, \mathbf{b})$ with $\mathbf{b} \in \mathbb{N}^r$.
 - ▶ The value ω is allowed in $\mathbf{b}$ in the initial configuration $(q_0, \mathbf{b})$.
- ▶ Parity game problem for AVASS:
 - Input:** AVASS $\mathcal{A}$, $q_0, \mathbf{b} \in (\mathbb{N} \cup \{\omega\})^r$ and $\text{col} : Q \rightarrow [0, p]$,
 - Question:** is there a proof with root $(q_0, \mathbf{b})$, all the maximal branches are infinite and the maximal colour that appears infinitely often along each branch is even?
- ▶ The parity game problem for AVASS is decidable.

Correspondences & Computational Complexity

Main correspondences

RB $\pm$ ATL	Alternating VASS
Logic in AI	Verification games
<i>proponent restriction condition</i>	updates in R_1 / no update in R_2
computation tree for F_A	proof
formulae in the scope of $\langle\langle A^b \rangle\rangle$	monotone objectives

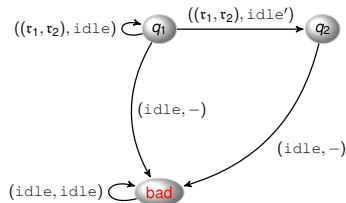
- ▶ From RB $\pm$ ATL model-checking to the state reachability and the non-termination problems for AVASS.
- ▶ From RB $\pm$ ATL* model-checking to the parity games for AVASS.
- ▶ Parameters synthesis thanks to the computation of the Pareto frontier of parity games.

See [Abdulla et al., CONCUR'13]

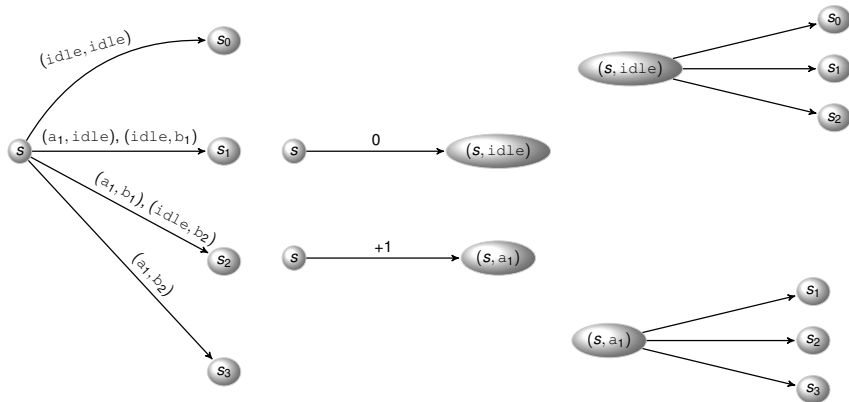
2EXPTIME-hardness

- ▶ Reduction from the state reachability problem for AVASS.
- ▶ Restriction to strict alternation:
 - ▶ $Q = Q_1 \uplus Q_2$ and $q_0, q_f \in Q_1$,
 - ▶ $R_1 \subseteq Q_1 \times \mathbb{Z}^r \times Q_2$ and $R_2 \subseteq Q_2 \times Q_1 \times \mathbb{Z}^r$,
 - ▶ there is no rule starting by q_f and there is a unary rule starting by q_0 .
- ▶ $\mathcal{A} = (Q, r, R_1, R_2) \mapsto \mathfrak{M} = (Agt, S, Act, r, act, cost, \delta, L)$
 s.t. $\mathfrak{M}, q_0 \models \langle \langle \{1\}^0 \rangle \rangle \top \mathcal{U} q_f$ iff there is a finite proof of AVASS with root $(q_0, \mathbf{0})$ and each leaf in $\{q_f\} \times \mathbb{N}^r$.

$$\tau_1 = q_1 \xrightarrow{(-1, +3)} q_0 \quad \tau_2 = q_0 \rightarrow q_1, q_2$$



From models to alternating VASS



$$Agt = \{1, 2\} \quad A = \{1\}$$

A more formal definition

$$\mathfrak{M} = (\text{Agt}, S, \text{Act}, r, \text{act}, \text{cost}, \delta, L) \mapsto \mathcal{A}_{\mathfrak{M}, A, s} \stackrel{\text{def}}{=} (Q, r, R_1, R_2)$$

► $Q\{s\} \cup \dots$

$$\{(s', f) \mid s' \in S, f \in D_A(s')\} \cup \{(g, s') \mid s', s'' \in S, g \in D(s''), \delta(s'', g) = s'\}$$

► Unary rules:

- for all $f \in D_A(s)$, $(s, \text{cost}_A(s, f), (s, f))$,
- for all $(g, s') \in Q$, for all $f \in D_A(s')$,
 $((g, s'), \text{cost}_A(s', f), (s', f))$.

► For all $(s', f) \in Q$, fork rule $((s', f), (g_1, s_1), \dots, (g_\alpha, s_\alpha))$ with

$$\{(g_1, s_1), \dots, (g_\alpha, s_\alpha)\} = \{(g, s'') \in Q \mid s'' \in \delta(s', g), g \in D(s'), f \sqsubseteq g\}$$

Main correspondences

- ▶ $\mathcal{A}_{\mathfrak{M},A,S'}^{S'}$: restriction of $\mathcal{A}_{\mathfrak{M},A,S}$ in which $(Agt \setminus A)$ has no way to go out of S' .
- ▶ Equivalent statements (assuming $s \in S'$):
 1. There is a **b**-strategy F_A w.r.t. s in $\mathfrak{M}$ s.t. $\text{out}(s, F_A)$ only visit states in S' .
 2. There is a proof in $\mathcal{A}_{\mathfrak{M},A,S}^{S'}$ from $(s, \mathbf{b})$ and every maximal branch is infinite.
- ▶ Equivalent statements ($S_1, S_2 \subseteq S$):
 1. There is a **b**-strategy w.r.t. s in $\mathfrak{M}$ s.t. $\text{out}(s, F_A)$ is “included in” $S_1^* S_2 S^\omega$.
 2. There is a finite proof in $\mathcal{A}_{\mathfrak{M},A,S}^{S_1 \cup S_2}$ from $(s, \mathbf{b})$ and each leaf contains a control state in

$$\{(g, s') \in Q \mid s' \in S_2\} \cup \{s' \in Q \mid s' = s, s \in S_2\}$$

Algorithm GMC($\mathfrak{M}, \phi$) using the correspondences

► p : return $\{s \in S \mid p \in L(s)\}$.

► $\neg\psi$: return $S \setminus \text{GMC}(\mathfrak{M}, \psi)$.

► $\langle\langle A^{\mathbf{b}} \rangle\rangle \bigcirc \psi$: return

$\{s \mid \exists f \in D_A(s), \mathbf{0} \preceq \mathbf{b} + \text{cost}_A(s, f), \forall f \sqsubseteq g \in D(s), \delta(s, g) \in \text{GMC}(\mathfrak{M}, \psi)\}$

► $\langle\langle A^{\mathbf{b}} \rangle\rangle \Box \psi$: $S_1 := \text{GMC}(\mathfrak{M}, \psi)$. If $s \notin S_1$ return $\emptyset$ otherwise return $\{s \mid \mathcal{A}_{\mathfrak{M}, A, s}^{S_1}(s, \mathbf{b}) \text{ is non-terminating}\}$.

► For $\langle\langle A^{\mathbf{b}} \rangle\rangle \psi_1 \mathcal{U} \psi_2$, use a procedure for the state reachability problem for AVASS.

Complexity upper bounds

- ▶ Model-checking problem for $\text{RB}\pm\text{ATL}$ in 2EXPTIME .
(hardness with $\text{card}(Agt) = 2$)
- ▶ For $r \geq 1$, the problem restricted to r resources in EXPTIME .
(consequence of MFCS'14 & ICALP'15 papers + our algo.)
- ▶ For $r \geq 4$, the problem is EXPTIME -hard.
(as a consequence of [Courtois & Schmitz, MFCS'14])
- ▶ $\text{card}(Agt) = 1$: EXPSPACE -completeness.
- ▶ For $r \geq 2$, the problem is PSPACE -hard.
(as a consequence of [Blondin & et al., LICS'15])
- ▶ For $r = 1$, the problem is NP -hard.
(as a consequence of [Haase, PhD 2012]).

The logic $\text{RB}\pm\text{ATL}^*$

- ▶ $\text{RB}\pm\text{ATL}^* = \text{RB}\pm\text{ATL} + \text{LTL-like path formulae}.$
- ▶ LTL-like path formulae handled with det. parity automata.
- ▶ Synchronized AVASS $\mathcal{A}_{\mathfrak{M},A,s} \otimes \mathbb{A}.$
- ▶ Equivalence between:
 - ▶ $\mathcal{A}_{\mathfrak{M},A,s}$ has a proof from $(s, \mathbf{b})$, all the maximal branches are infinite and the L -projection of each infinite branch belongs to $L(\mathbb{A})$.
 - ▶ $\mathcal{A}_{\mathfrak{M},A,s} \otimes \mathbb{A}$ has a proof from $((s, q'_0), \mathbf{b})$, all the maximal branches are infinite and the maximal colour that appears infinitely often is even.
- ▶ The model-checking problem for $\text{RB}\pm\text{ATL}^*$ is decidable.

(the method generalises what is done for ATL^*)

Concluding remarks

- ▶ Formal relationships between resource-bounded logics and games on alternating VASS.
- ▶ The MC problem for $RB\pm ATL$ is $2EXPTIME$ -complete.
- ▶ The MC problem for $RB\pm ATL^*$ is decidable ...
but **MC for CTL^* on VASS is undecidable.**
- ▶ The parameterized version admits effective computations of the parameters. (not presented in the talk)
- ▶ $EXPSpace$ -completeness for $RBTL$ and $RBTL^*$.
(not presented in the talk)