TD 8

02/12/2009

Exercise 1 (TATA 8.3)**.** Give direct construction on hedge automata that show they are closed under boolean operations (complementation, union, intersection).

Exercise 2 (TATA 8.1)**.** We define *Yield* on unranked trees similarly as for ranked trees ($Yield(t)$ returns a word containing exactly the leafs of t in the order they appear in t).

Let R be a regular word language. Construct a hedge automaton recognizing $\{t \mid Yield(t) \in R\}$.

Exercise 3 (TATA 8.4)**.** Consider a ranked signature $\mathcal{F}$ and an unranked signature Σ which is obtained from $\mathcal{F}$ by dropping the arities. Let $f \in \mathcal{F}$ be a binary function symbol. We will consider that f represents an associative function.

We define the transformation A which turns a ranked tree into an unranked tree by "gathering" all the arguments of nested applications of f into a single node also labeled f. Formally:

$$\begin{array}{rcll} A(g(t_1,\ldots,t_n)) & = & g(A(t_1),\ldots,A(t_n)) & \text{if } g \neq f \\ A(f(t_1,t_2)) & = & f(A_f(t_1),A_f(t_2)) & \\ A_f(f(t_1,t_2)) & = & A_f(t_1)A_f(t_2)) & \text{this is a hedge} \\ A_f(g(t_1,\ldots,t_n)) & = & g(A(t_1),\ldots,A(t_n)) & \text{if } g \neq f \end{array}$$

1. Give a regular language $L \subseteq T(\mathcal{F})$ such that $A(L) = \{A(t) \mid t \in L\}$ is not regular.

2. Show that if $L \subseteq T(\mathcal{F})$ is closed under the congruence defined by

$$f(x, f(y, z)) = f(f(x, y), z)$$

then $A(L)$ is regular.