# TD 7

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**Exercise 1.** 1. Recall what is the form of the production rules $R$ if $G = (S, N, T, R)$ is a context-free *word* grammar, where $S \in N$ is the axiom, $N$ is the set of non-terminals and $T$ is the set of terminals.

2. Let $G = (S, N, T, R)$ be a context-free word grammar. Consider the signature $\mathcal{F}_G$ consisting of the constants (function symbols with arity 0) $T$ and of the function symbols $(N, n)$ of arity $n$ for each production rule of the form $N \to w_1 \dots w_n$.

   Let $G_0 = (S, \{S, O\}, \{x, +, *\}, R)$ be a context-free word grammar, where

$$R = \{ \begin{array}{l} S \to S\ O\ S \\ S \to x \\ O \to + \\ O \to * \end{array} \quad \}$$

   What is $\mathcal{F}_{G_0}$?

3. The set of parse trees $P_G(a)$ associated with a symbol in $a \in T \cup N$ is the smallest set such that:

   (a) $P_G(a) = \{a\}$ if $a \in T$

   (b) $(a, n)(t_1, \dots, t_n) \in P_G(a)$ if $t_1 \in P_G(a_1), \dots, t_n \in P_G(a_n)$ and $a \to a_1 \dots a_n \in R$ $(n \geq 0)$

   The set of parse trees of $G$ is $P_G = P_G(S)$.

   Write a parse tree of the word $x * x + x$.

4. Let $G$ be a context-free word grammar. Prove that $P_G$ is a regular tree language.

5. If $t$ is a tree/term, we define *Leaves*$(t)$ inductively as follows:

   (a) $\mathit{Leaves}(f) = f$, if $f \in \mathcal{F}_0$

   (b) $\mathit{Leaves}(f(t_1, \dots, t_n)) = \mathit{Leaves}(t_1) \dots \mathit{Leaves}(t_n)$

By notation, let $S_n \doteq (S, n)$ and $O_1 \doteq (O, 1)$. "Compute"

$$Leaves(S_3(S_1(x), O_1(*), S_3(S_1(x), O_1(+), S_1(x))))$$

6. Let $L$ be a regular tree language. Prove that $Leaves(L) \stackrel{def}{=} \bigcup_{t \in L}(Leaves(t))$ is a context-free language.

7. Show that there exists a regular tree language which is not the set of parse trees of a context-free language.

**Exercise 2.** Define the following languages in WSkS:

1. the set of terms of height at least 3,

2. the set of terms whose branches all have length at least 3,

3. the set of terms that contain an even number of the $f$ function symbol.

**Exercise 3.** Show that the relation $\{(f(t, t'), t) \mid t, t' \in \mathcal{T}(\mathcal{F})\}$ is not definable in WS2S.