TD 5

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Exercise 1. Let t be a ground term. The path language of t, denoted $\pi(t)$, is the *word language* defined as follows:

- if $t \in \mathcal{F}_0$, then $\pi(t) = t$
- if $t = f(t_1, \ldots, t_n)$, then $\pi(t) = \bigcup_{i=1}^{n} \{fiw \mid w \in \pi(t_i)\}$

If L is a tree language, the path language of L, denoted $\pi(L)$, is defined to be

$$\pi(L) = \bigcup_{t \in L} \{\pi(t)\}$$

The pathclosure of L, denoted *pathclosure*(L), is defined to be

$$\mathit{pathclosure}(L) = \{t \mid \pi(t) \subseteq \pi(L)\}$$

A tree language L is *path-closed* if *pathclosure*$(L) = L$.

1. Recall the language $L_0 = \{f(a, b), f(b, a)\}$. Recall the proof that L_0 is not recognized by a top-down DFTA.
2. What is $\pi(L_0)$?
3. What is *pathclosure*(L_0)?
4. Is *pathclosure*(L_0) regular? Is it recognized by a top-down DFTA?
5. Prove that if L is a recognizable tree language, then $\pi(L)$ is a recognizable word language.
6. Prove that if L is a recognizable tree language, then *pathclosure*(L) is a recognizable tree language.
7. Prove that a recognizable tree language is path-closed iff it is recognizable by a top-down DFTA.
8. Is it decidable whether a regular tree language defined by a NFTA is path-closed?

Exercise 2. 1. Recall what is the form of the production rules R if $G = (S, N, T, R)$ is a context-free *word* grammar, where $S \in N$ is the axiom, N is the set of non-terminals and T is the set of terminals.

2. Let $G = (S, N, T, R)$ be a context-free word grammar. Consider the signature $\mathcal{F}_G$ consisting of the constants (function symbols with arity 0) T and of the function symbols (N, n) of arity n for each production rule of the form $N \to w_1 \dots w_n$.

   Let $G_0 = (S, \{S, O\}, \{x, +, *\}, R)$ be a context-free word grammar, where

$$R = \{ \begin{array}{l} S \to S\ O\ S \\ S \to x \\ O \to + \\ O \to * \end{array} \}$$

   What is $\mathcal{F}_{G_0}$?

3. The set of parse trees $P_G(a)$ associated with a symbol in $a \in T \cup N$ is the smallest set such that:

   (a) $P_G(a) = \{a\}$ if $a \in T$

   (b) $(a, n)(t_1, \dots, t_n) \in P_G(a)$ if $t_1 \in P_G(a_1), \dots, t_n \in P_G(a_n)$ and $a \to a_1 \dots a_n \in R$ $(n \geq 0)$

   The set of parse trees of G is $P_G = P_G(S)$.

   Write a parse tree of the word $x * x + x$.

4. Let G be a context-free word grammar. Prove that P_G is a regular tree language.

5. If t is a tree/term, we define $\mathit{Leaves}(t)$ inductively as follows:

   (a) $\mathit{Leaves}(f) = f$, if $f \in \mathcal{F}_0$

   (b) $\mathit{Leaves}(f(t_1, \dots, t_n)) = \mathit{Leaves}(t_1) \dots \mathit{Leaves}(t_n)$

   By notation, let $S_n \doteq (S, n)$ and $O_1 \doteq (O, 1)$. "Compute"

$$\mathit{Leaves}(S_3(S_1(x), O_1(*), S_3(S_1(x), O_1(+), S_1(x))))$$

6. Let L be a regular tree language. Prove that $\mathit{Leaves}(L) \stackrel{def}{=} \bigcup_{t \in L}(\mathit{Leaves}(t))$ is a context-free language.

7. Show that there exists a regular tree language which is not the set of parse trees of a context-free language.