

$$\text{TD } 2 \setminus \{6\} =_{\alpha} \text{TD } 1 \setminus \{1, 2, 4\}$$

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Exercise 1 (TATA 1.2). Let $\mathcal{F} = \{f(\cdot), g(\cdot), a\}$. Define a top-down NFTA, a NFTA and a DFTA for the set $M(t)$ of terms which contain a ground instance of the term $t = f(a, g(x))$ as a subterm (i.e. $M(t) = \{C[f(a, g(u))] \mid C \in \mathcal{C}(\mathcal{F}), u \in T(\mathcal{F})\}$). Is it possible to define a top-down DFTA for this language? A finite union of top-down DFTA?

Exercise 2 (TATA 1.5). Prove the equivalence between top-down NFTA and NFTA.

Exercise 3 (TATA 1.9). Let t be a linear term. Prove that $G(t)$ is recognizable. Give an example of a non-linear term t such that $G(t)$ is not recognizable.

Exercise 4 (TATA 1.15). The *complement problem* is the following:

Instance: $t, t_1, \dots, t_n \in T(\mathcal{F}, \mathcal{X})$.

Question: Is there a term s such that $s \in G(t)$, but $s \notin G(t_i)$ ($1 \leq i \leq n$)?

Prove that this problem is decidable whenever t, t_1, \dots, t_n are linear.

Exercise 5. Compute a tree automata which accepts the set of ground terms over $\mathcal{F} = \{f(\cdot), g(\cdot), a\}$ which are irreducible with respect to the following term rewriting system:

$$f(g(x), a) \rightarrow a \quad g(f(x, y)) \rightarrow y \quad f(f(x, y), z) \rightarrow z$$

Exercise 6. Design a linear time algorithm to decide the emptiness of a tree automaton.