TD 11

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Exercise 1. We consider finite ordered unranked trees and some predicates P_i on nodes. A boolean conjunctive query is a first-order formula of the following form:

$$\exists \bar{x}. A_1 \wedge \ldots \wedge A_n$$

where A_i are atomic formulae: either $A_i = P_j(x)$ or $A_i = x \leq y$ for some binary relation $\leq$ among a fixed set of relations ($\leq$ may be one of the child, next-sibling, etc). Furthermore all variables which appear in $A_1, \ldots, A_n$ must be in $\bar{x}$.

1. Show that if the allowed binary relations are *child* (x *child* y iff y is a child of x) and *child** (x *child** y iff y is a descendant of x), then the complexity of evaluating a query over a tree is NP-complete (even if the tree is fixed).

   Hint: reduce from monotone 1-in-3 3-SAT.

2. Show that if only the *child** relation is allowed, then the complexity of evaluating a query over a tree is in PTIME.