TD 1

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Exercise 1. Let $\mathcal{F} = \{f(,), a, b\}$. Show that $L = \{f(a,b), f(b,a)\}$ is not recognizable by a top-down DFTA.

Exercise 2 (TATA 1.1)**.** Let $\mathcal{F} = \{f(,), g(), a\}$. Define a top-down NFTA, a NFTA and a DFTA for the set $G(t)$ of ground instances of the term $t = f(f(a,x), g(y))$ (i.e. $G(t) = \{f(f(a,u), g(v)) \mid u, v \in T(\mathcal{F})\}$). Is it possible to define a top-down DFTA for this language?

Exercise 3 (TATA 1.2)**.** Let $\mathcal{F} = \{f(,), g(), a\}$. Define a top-down NFTA, a NFTA and a DFTA for the set $M(t)$ of terms which contain a ground instance of the term $t = f(a, g(x))$ as a subterm (i.e. $M(t) = \{C[f(a, g(u))] \mid C \in \mathcal{C}(\mathcal{F}), u \in T(\mathcal{F})\}$). Is it possible to define a top-down DFTA for this language? A finite union of top-down DFTA?

Exercise 4 (TATA 1.3)**.** Let $\mathcal{F} = \{g(), a\}$. Is the set of ground terms over $\mathcal{F}$ whose height is even recognizable? Let $\mathcal{F}' = \{f(,), g(), a\}$. Is the set of ground terms over $\mathcal{F}'$ whose height is even recognizable?

Exercise 5 (TATA 1.5)**.** Prove the equivalence between top-down NFTA and NFTA.

Exercise 6 (TATA 1.9)**.** Let t be a linear term. Prove that $G(t)$ is recognizable. Give an example of a non-linear term t such that $G(t)$ is not recognizable.

Exercise 7 (TATA 1.15)**.** The *complement problem* is the following:
Instance: $t, t_1, \ldots, t_n \in T(\mathcal{F}, \mathcal{X})$.
Question: Is there a term s such that $s \in G(t)$, but $s \notin G(t_i)$ $(1 \leq i \leq n)$?
Prove that this problem is decidable whenever $t, t_1, \ldots, t_n$ are linear.

Exercise 8. Compute a tree automata which accepts the set of ground terms over $\mathcal{F} = \{f(,), g(), a\}$ which are irreducible with respect to the following term rewriting system:

$$f(g(x), a) \to a \qquad g(f(x,y)) \to y \qquad f(f(x,y), z) \to z$$