



Axe Infini

Algorithmics for symbolic verification of infinite systems



Alain FINKEL

Plan

1. Introduction

1. Scientific context
2. Forward reachability procedure
3. Backward reachability algorithm
4. Our two preferred models

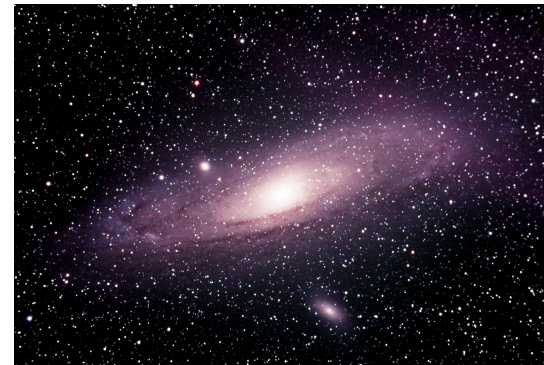
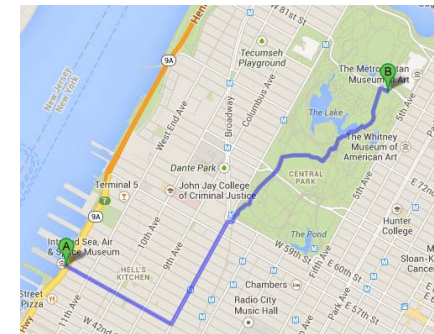
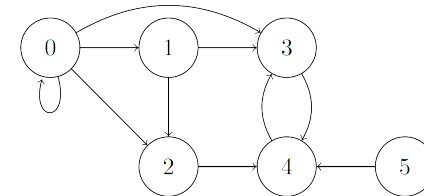
2. WSTS and CM

3. Management

4. 5 Strategy

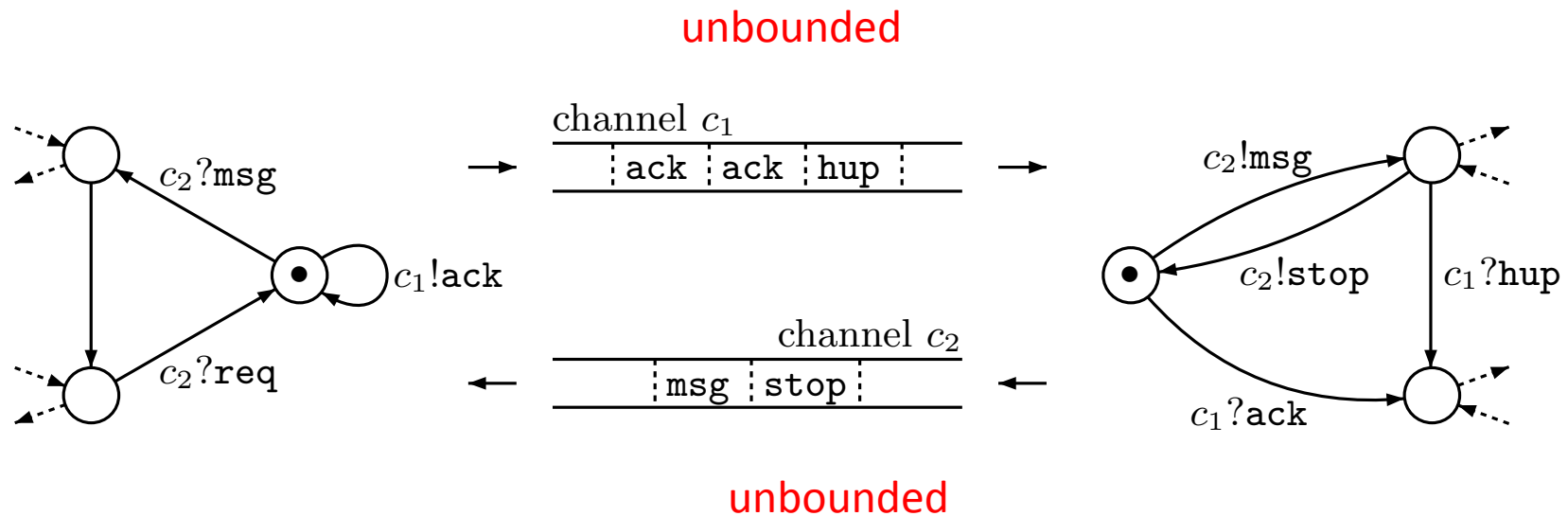
Scientific context

- What objectives do we address ?
 - Automatic analysis of (abstraction of) programs with unbounded parameters (stack, channel, recursive,...).
 - Analyse = solve reachability from s towards t
 - Analyse = $\exists \text{ path } s \xrightarrow{*} t$ in an infinite graph ?
 - Needs finite and computable representations



Communicating programs

finite control but unbounded channels



Lists program

is this program correct ?

```
List Reverse (List x) {  
    List y,t;  
    y= NULL;  
    while (x!=NULL) {  
        t=y;  
        y=x;  
        x=x->n;  
        y=n->t;  
        t=NULL;}  
    return y;}  
}
```

Recursivity and counter

- From $P(17)$, will R necessarily be activated ?

$P(x)$: If $x \geq 16$

 If $8|x$ then $Q(x + 1)$
 else $P(x - 2)$

else $F(x)$

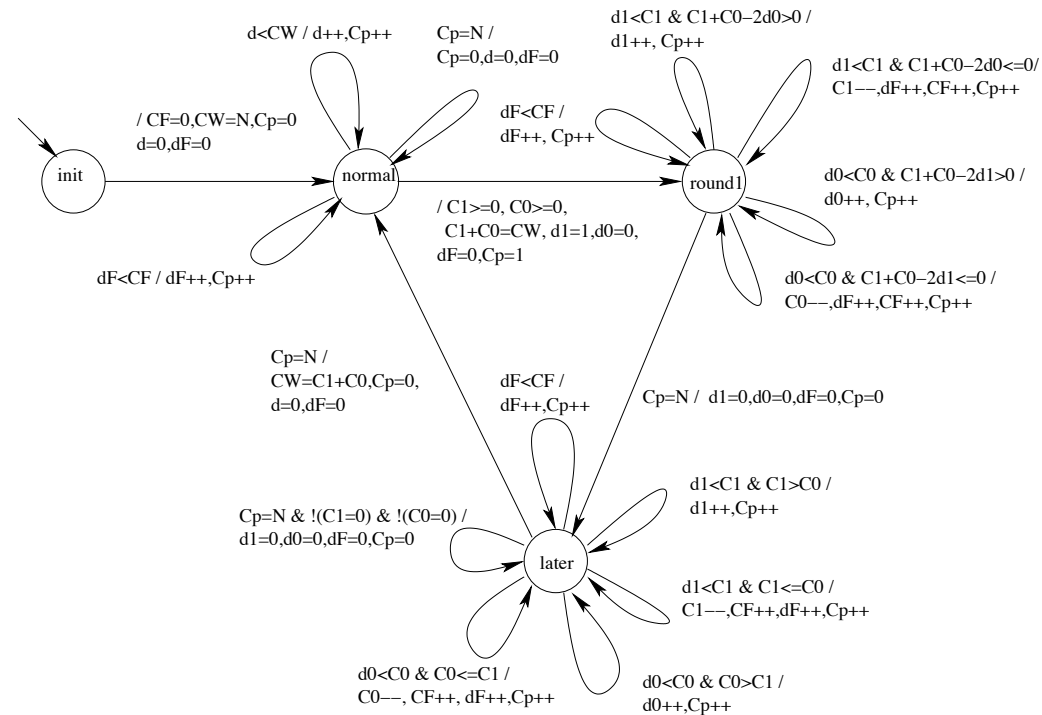
$Q(x)$: If $2|x$ then $R(x)$
 else $S(x + 1)$

In Bouajjani & al TCS n°295, 2003, page 86

Modelisation of the communication protocol TTP

only +1 and -1 operations but already difficult analysis

Model for the TTP, N stations



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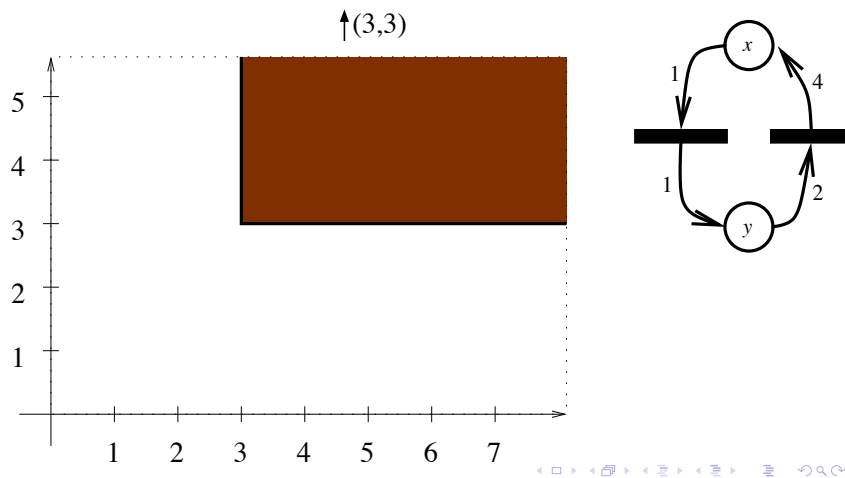
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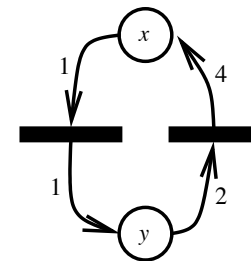
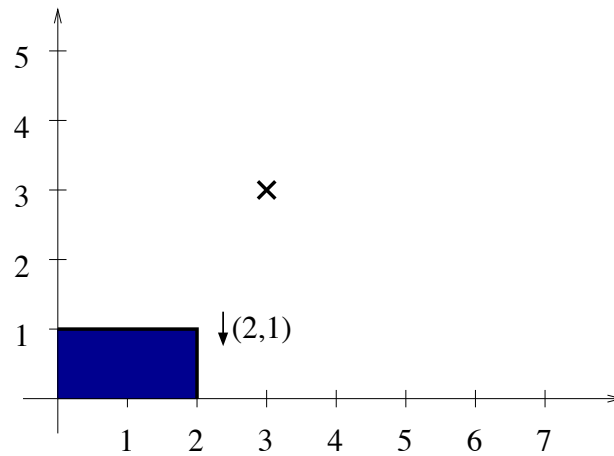
Reachability is difficult for “simple” CM

- $t_1: (x,y) \rightarrow (x-1,y+1)$
- $t_2: (x,y) \rightarrow (x+4,y-2)$

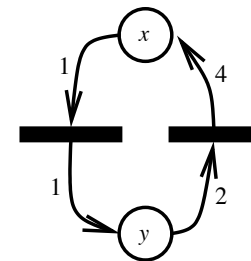
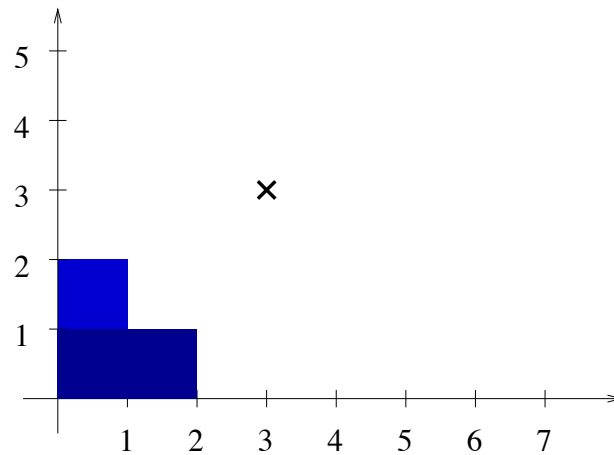


- Is there a path s.t.:
 $(2,1) \rightarrow \dots * \dots \rightarrow \uparrow(3,3)$?

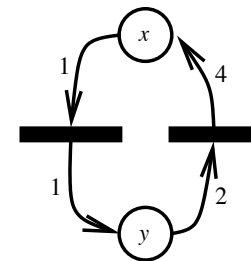
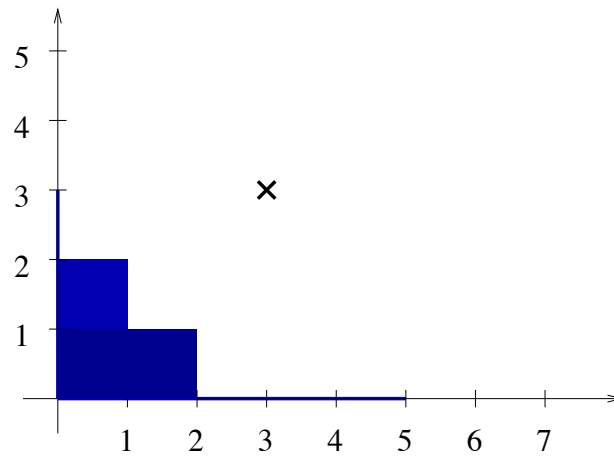
A Naive Forward Algorithm



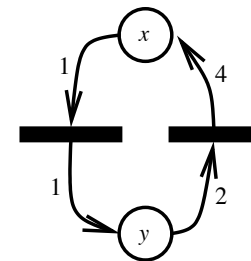
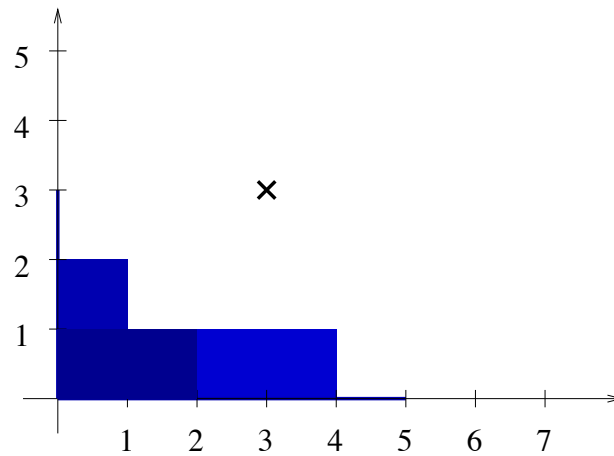
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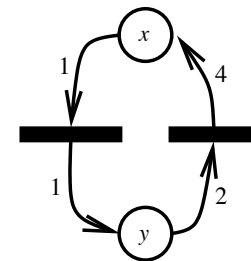
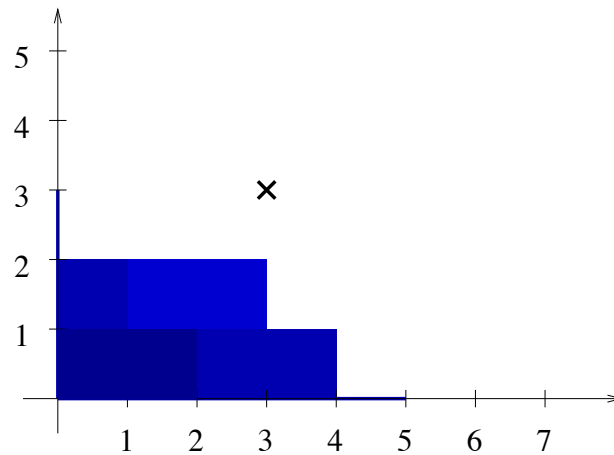
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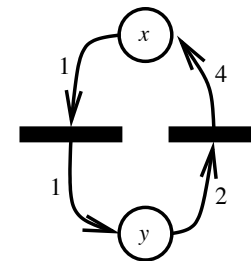
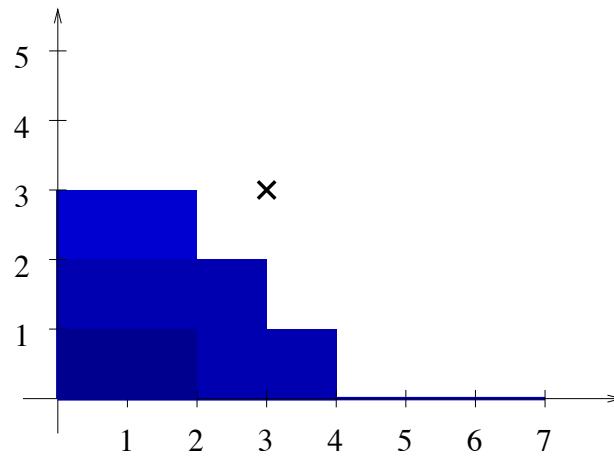
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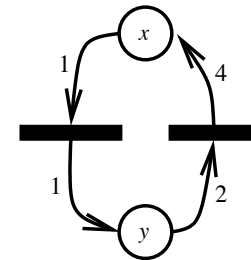
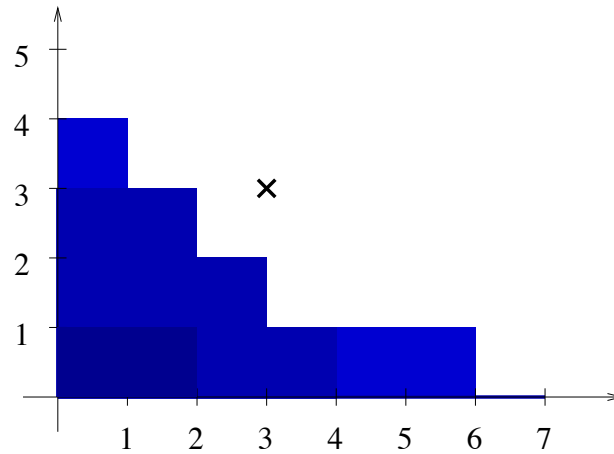
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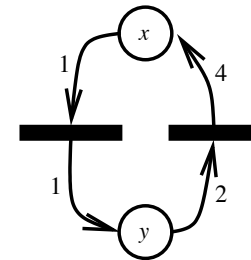
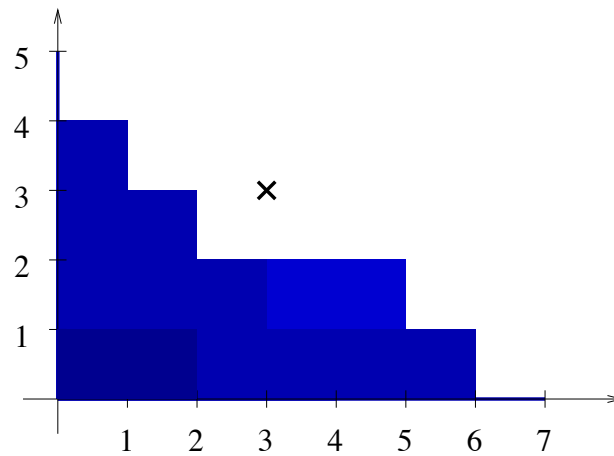
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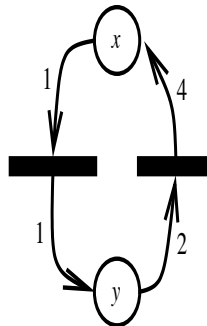
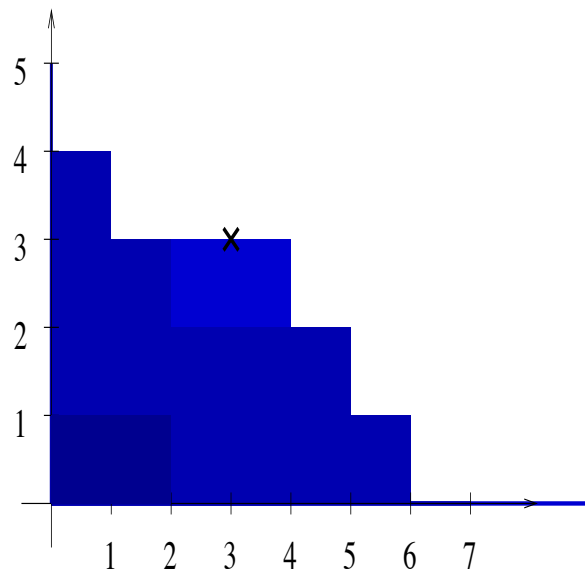
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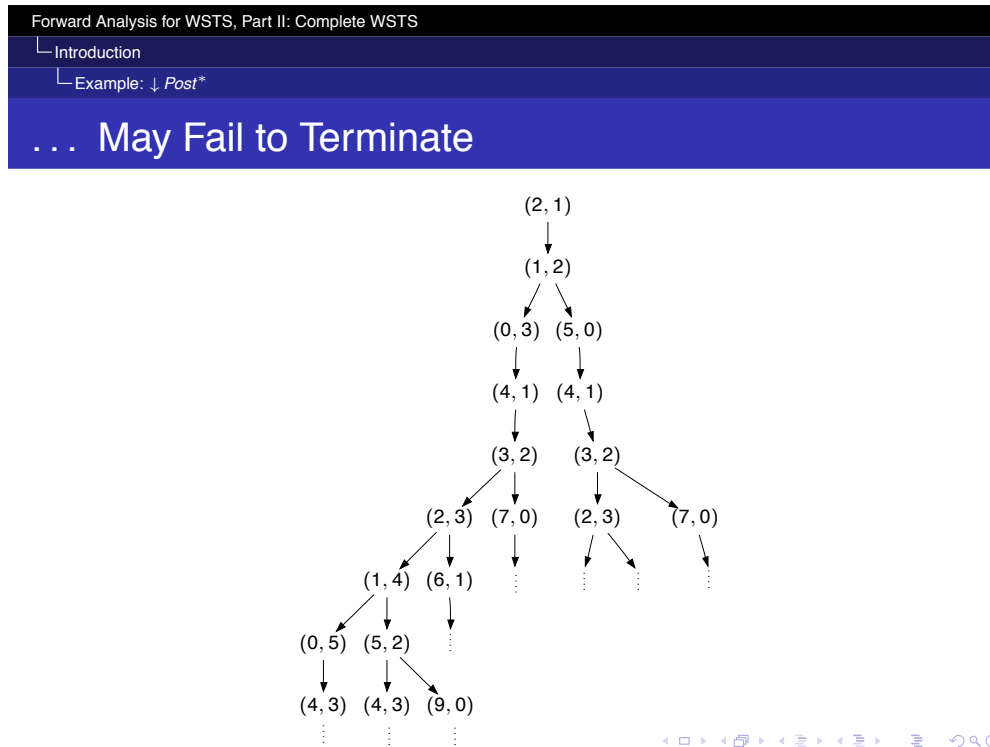


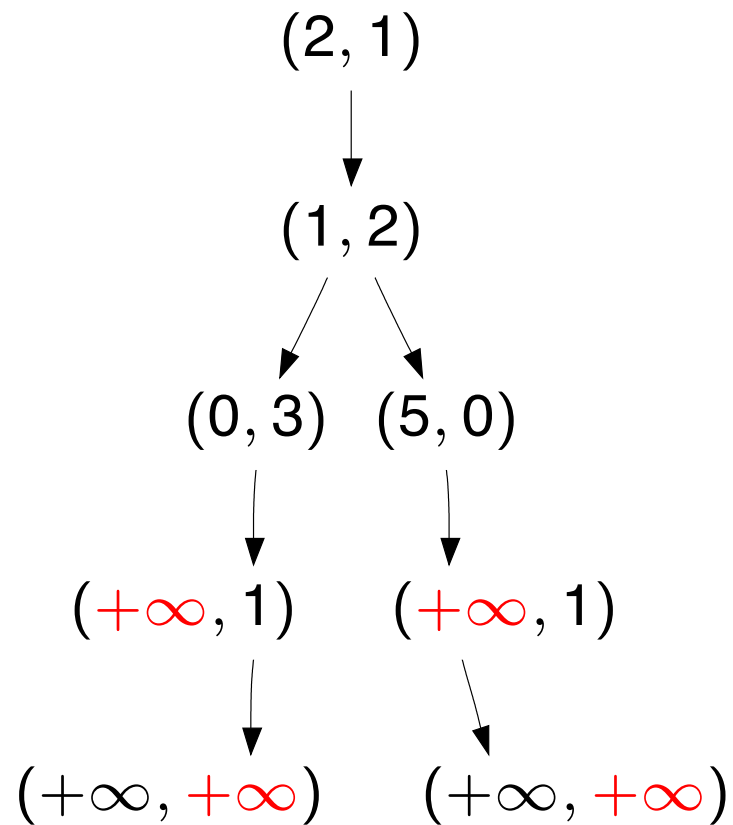
A Naive Forward Algorithm



- A path of length 8 from $\downarrow(2,1)$ which meets $(3,3)$

Infinite reachability tree





Is $\text{Post}^*(2,1)$ (finitely) computable ?
When does this procedure terminate ?

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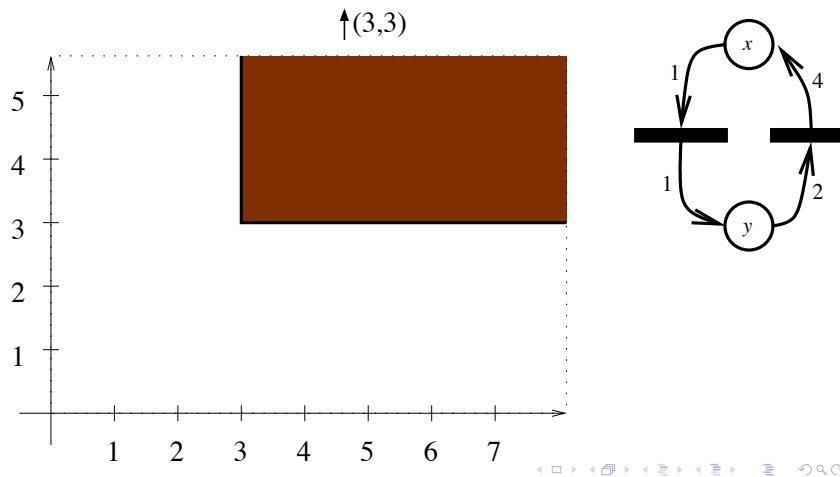
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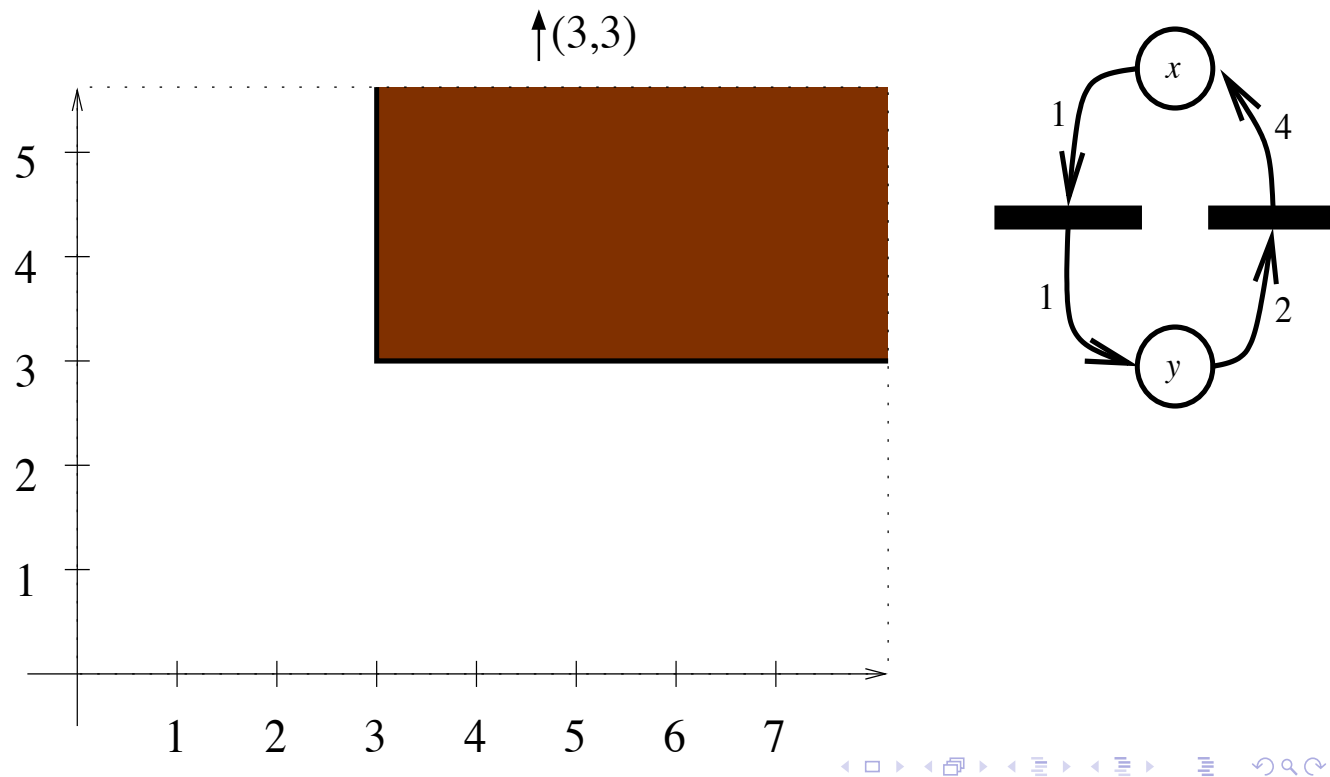
Reachability/coverability

- $t_1: (x,y) \rightarrow (x-1,y+1)$
- $t_2: (x,y) \rightarrow (x+4,y-2)$

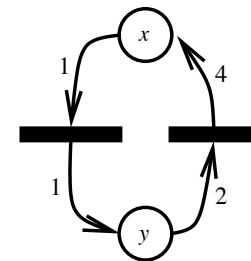
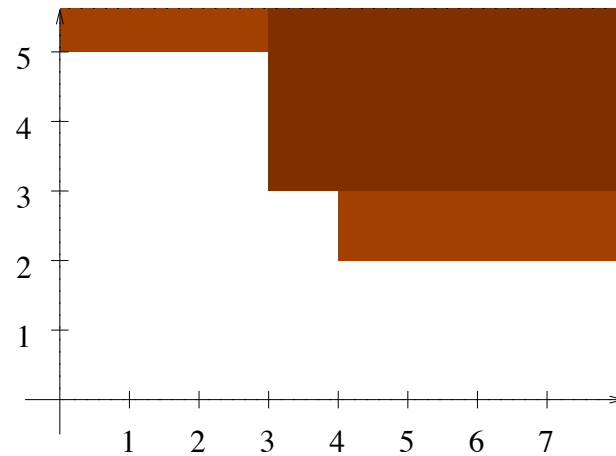


- Is there a path s.t.:
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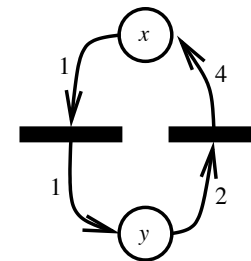
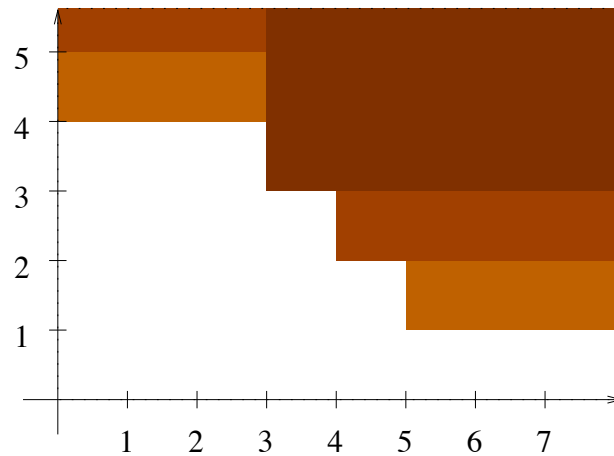
Example: Coverability (in Petri Nets, here)



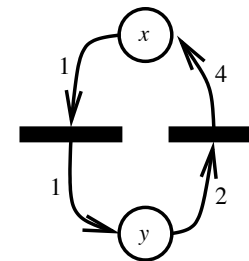
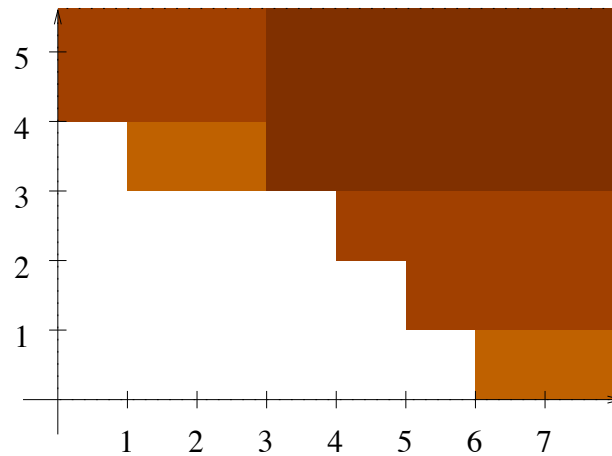
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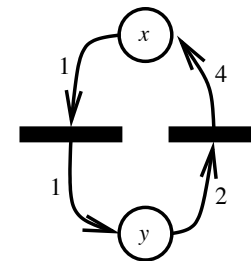
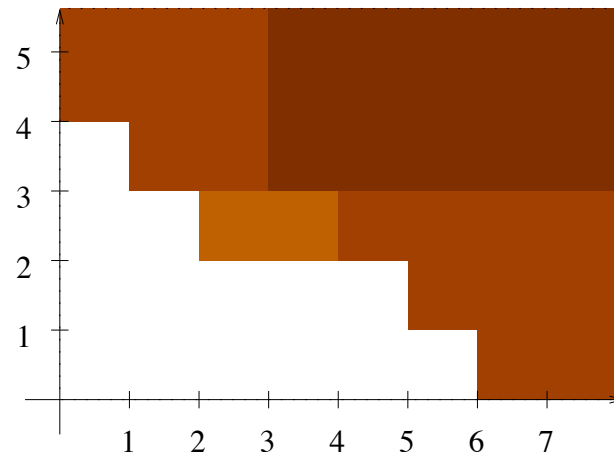
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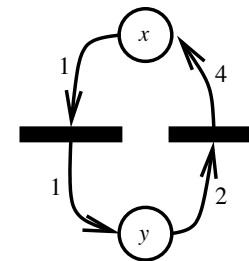
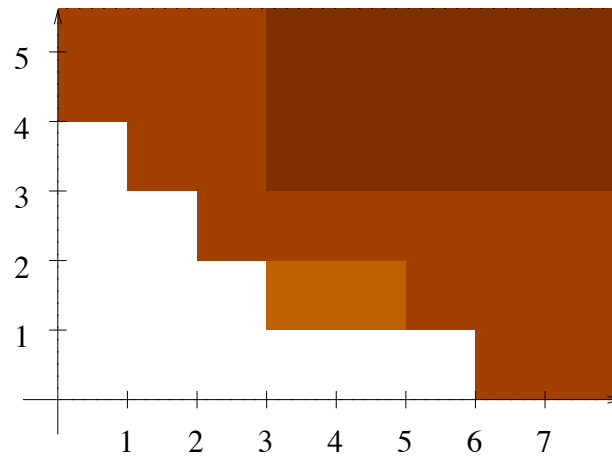
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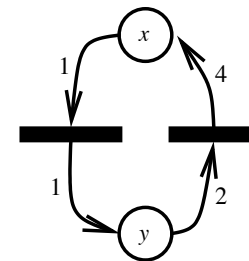
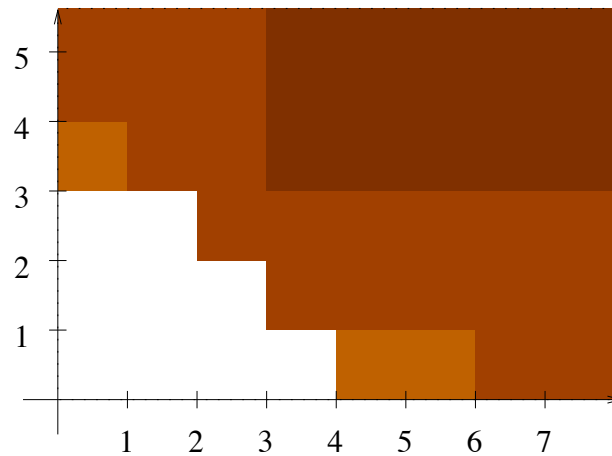
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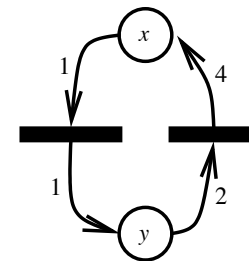
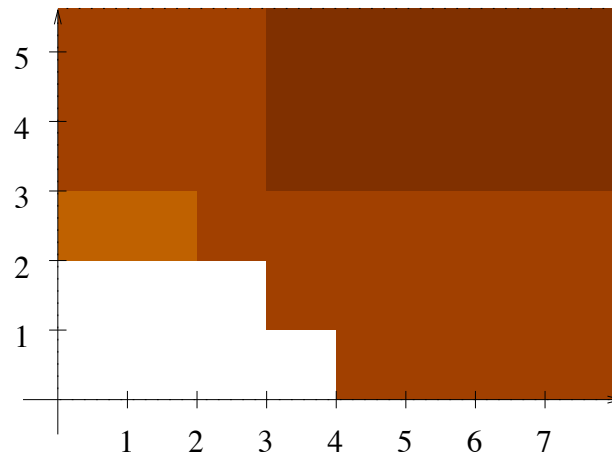
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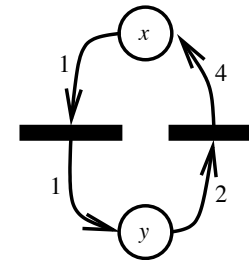
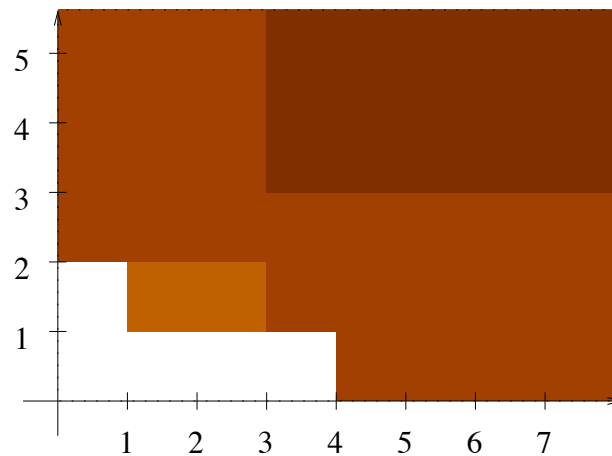
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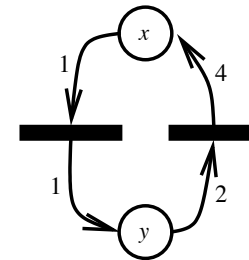
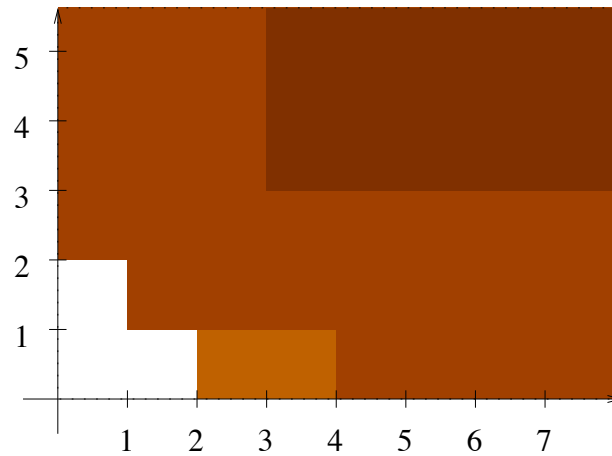
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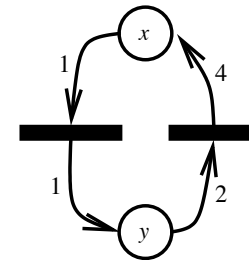
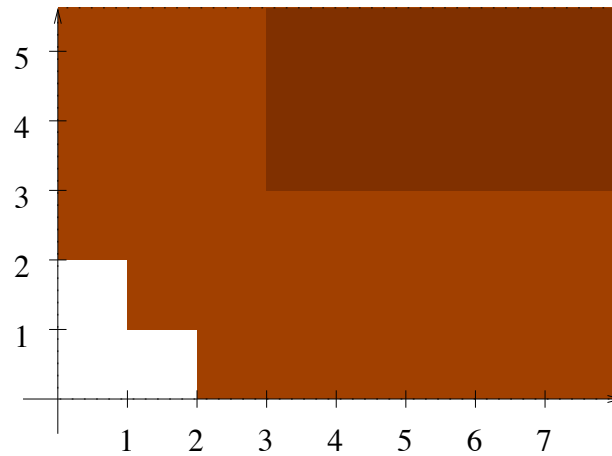
Example: Coverability (in Petri Nets, here)



Example: Coverability (in Petri Nets, here)



Example: Coverability (in Petri Nets, here)



Backward strategy

- A path of length 10 from $\uparrow (3,3)$ that finally contains state $(2,1)$.

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4. 5 Strategy

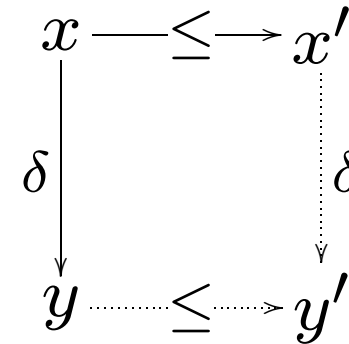
Our two preferred models

- WSTS: **generic** (=abstract) model for tens of « concrete » models
- Counter machines : **universel** model for computation (& modelization/verification)

Well Structured Transition Systems (WSTS)

■ Ingredients:

- A transition relation $\delta \subseteq X \times X$;
- A **well quasi-ordering** (wqo) $\leq$ on X ;
- + **monotonicity**:



■ $\leq$ is **wqo** iff (equivalently):

- no infinite descending chain, and no infinite antichain;
- every sequence has an infinite non-decreasing subsequence;
- every upward-closed subset U is of the form $\uparrow A$, A **finite**.

WSTS and WQO history

- Timeline

- ICALP 1987 : basis of the theory
- TCS 2001 : survey of the theory
- IPL 2002: Reachability is non Recursive Primitive
- STACS 2009 + ICALP 2009: mathematical foundations to forward analysis

- LICS 2008
- MFCS 2010
- LICS 2011
- ESSLLI 2012
- LICS 2012
- Petri 2011, Petri 2012
- CONCUR 2013

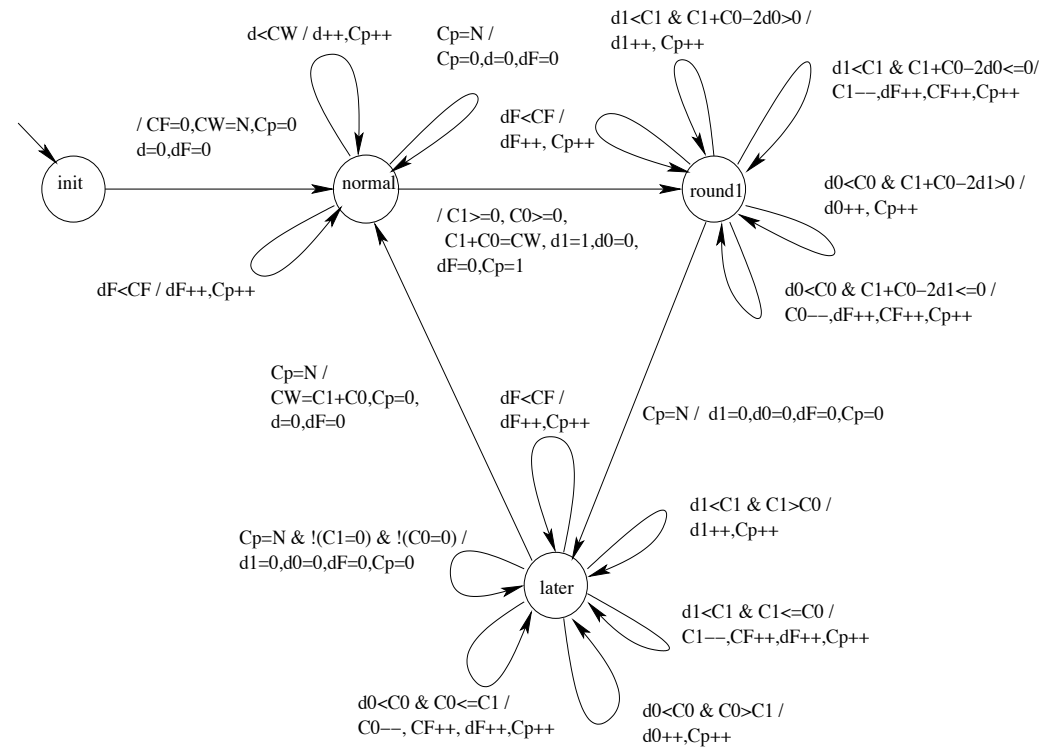


Complexity & decidability of WSTS

- Success : used and studied by numerous colleagues (Henzinger, Raskin,...)

Counter Machines (CM)

Model for the TTP, N stations



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1. Highlight: wqos and complexity
2. WSTS
3. Counter Machines

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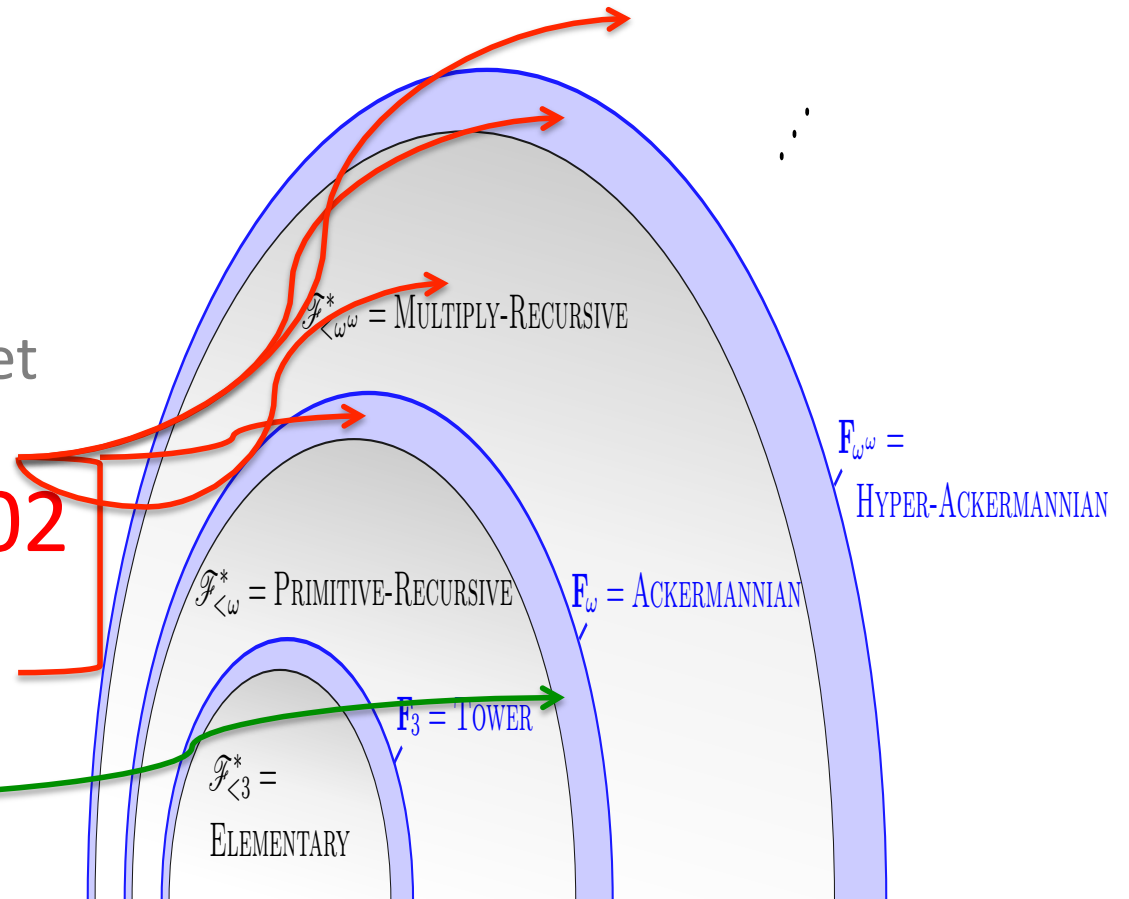
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Highlight : complexity of WQO

- Jancar TCS'01
The finite reachability set
equality for Petri nets

- **Schnoebelen IPL'02**
Reachability LCS

- Reachability LCM
(MFCS 2010)



Highlight : complexity of WQO

Challenges: Complexity of WSTS

- Concepts for measuring length of wqo (done)
- Upper/lower bounds (beginning)
- Publications LICS 2008, MFCS 2010, LICS 2011, FOSSACS 2011, ICALP 2011, LICS 2012, MFCS 2013

Projects 2013-2017

- Delineate models/problems
- Equivalences/hierarchy
- Towards a Garey & Johnson of non-elementary complexities

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A paradox ?

- Backward strategy always terminates but is not efficient.
- The tool Trex (LIAFA) does not use the backward algorithm

No paradox

Forward is more efficient than backward

- Backward strategy always terminates but is not efficient.
- The tool Trex (LIAFA) does not use the backward algorithm
- Accelerated forward strategy often terminates with efficiency but no theory
- Trex uses an adhoc accelerated forward procedure without termination guarantee

No theory of downward closed sets

Upward closed sets

- A nice and simple theory

- **Theorem** (~1900)

For all $X \subseteq A$, A wqo,

$\uparrow X = \uparrow \{x_1, x_2, \dots, x_n\}$ with $x_i \in X$.

Downward closed sets

- Missing theory

"Finally, we aim at developing generic methods for building downward closed languages, This would give a general theory for forward analysis of infinite state systems, in the same way the work is for backward analysis." Abdulla & [FORMATS 2004]

- **Open problem** (2008)

For all $X \subseteq A$, A wqo, is $\downarrow X$ finitely describable ?

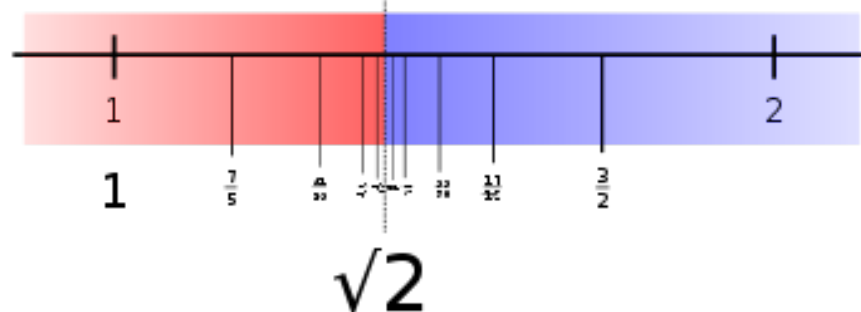
We have : $\downarrow X \neq \downarrow \{x_1, x_2, \dots, x_n\}$ with $x_i \in X$

Towards a theory of downward closed sets

Mathematical and cognitive hypothesis

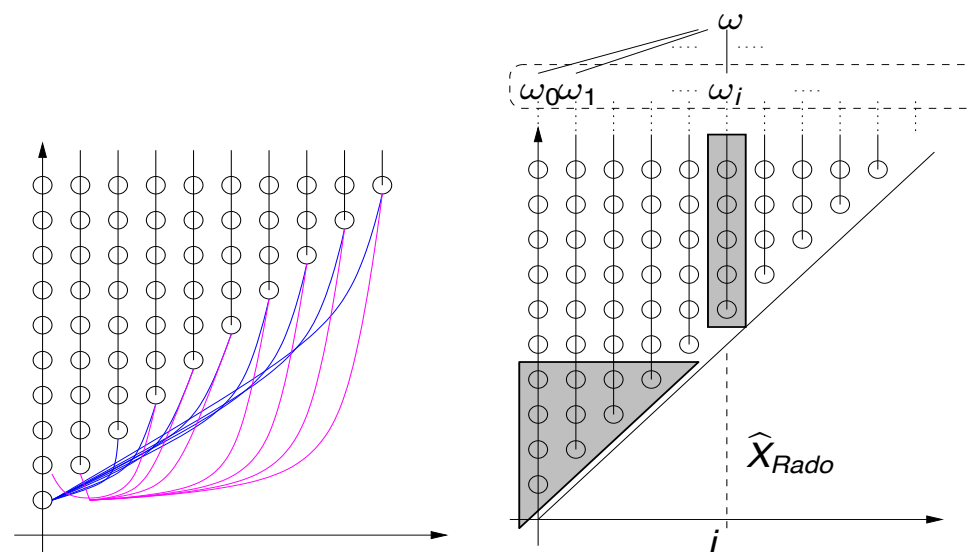
- limits are points at infinity but it does not work well
- Need of a « new » paradigm of limits which allows representing infinite downward closed sets.

Think « geometrically »



Think « algebraically »

- Identify $x \in X$ with the subset $\downarrow x \subseteq X$
- Elements and limits **are** directed downward closed subsets (= ideals)
- Theorem (STACS 2009)
 $\downarrow X = \{I_1, I_2, \dots, I_n\}$
 where I_i are ideals.



WSTS: challenges and results

2008 WSTS Challenges

- Build an unified general theory of forward analysis
- Simplify algorithms + proofs
- Measure the complexity

Results: 2008 - 2013

- Built a theory of downward closed sets
- Define Complete WSTS
- Define Completion of WSTS
- Conceptual & simple Karp&Miller algo
- **Measure the complexity (highlight)**
- Publications: **STACS 2009**, **ICALP 2009**, FOSSACS 2011, PETRI 2011, PETRI 2012

Results and projects

Results 2008 - 2013

- Build a theory of downward closed sets in wqo
- Complete WSTS
- Completion of WSTS
- Conceptual Karp&Miller algo
- Publications: STACS 2009, ICALP 2009, FOSSACS 2011, PETRI 2011, PETRI 2012

Projects 2013 - 2017

- Discover the good data-structures for ideals and completion-based algorithms
- Complexity of completion-based algorithms
- Ex: Priority channel systems

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Counter Machines

Challenges and results

CM Challenges in 2008

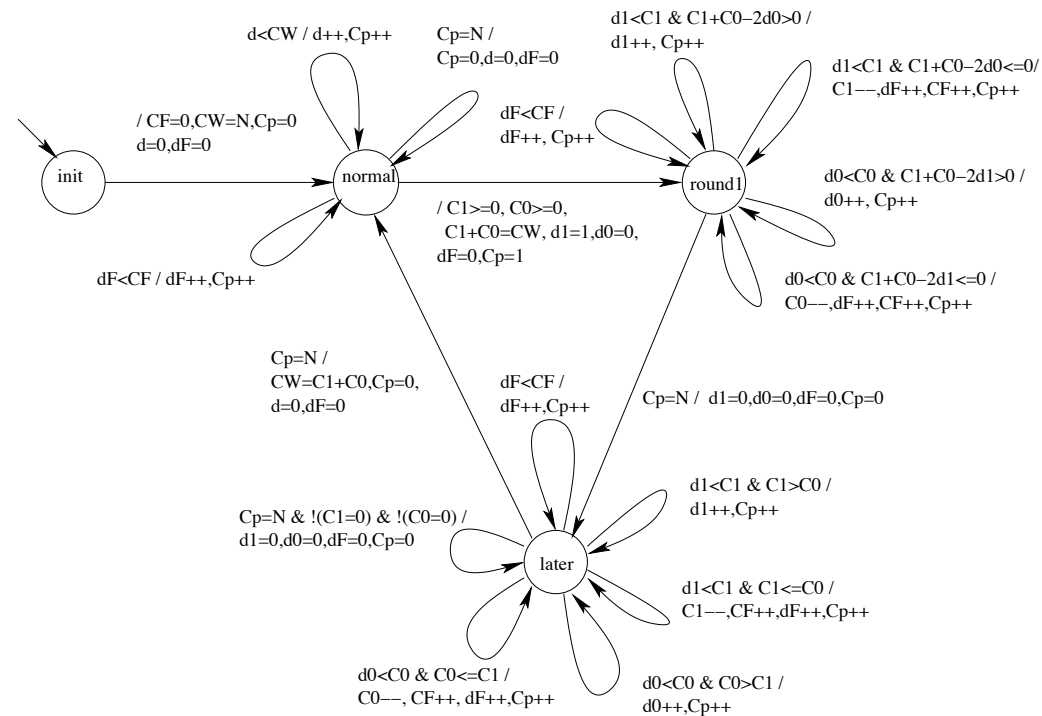
- Unify, extend complexity analysis of Petri nets and classes of CM.
- Complexity of coverability in Petri nets
- Complexity of reachability in Petri nets

CM results 2008 - 2013

- Complexity of logics for Petri nets, flat CM and reversal-bounded CM
- Large classes of “coverability-like” properties are EXPSPACE-complete on Petri nets
- **Publications** MFCS 2008, MFCS 2010, FSTTCS 2010, FSTTCS 2012, MFCS 2013, ICALP 2013, LICS 2013

Analysis by « simple » subsystems

Model for the TTP, N stations



Results and projects

CM results 2008 - 2013

- Complexity of logics for Petri nets, flat CM and reversal-bounded CM
- Large classes of “coverability-like” properties are EXPSPACE-complete on Petri nets
- Publications MFCS 2008, MFCS 2010, FSTTCS 2010, FSTTCS 2012, MFCS 2013, ICALP 2013, LICS 2013

Projects 2013 - 2017

- Complexity of reachability for Petri nets

EXSPACE PR ACK
Multiple-ACK $F_{\varepsilon 0}$

- Complete the book on CMs !
- Towards an algorithmics

Many other subjects...not mentioned

- Games (L. Doyen)
- Lossy channel systems (P. Schnoebelen)
- Parikh automata (with P. McKenzie, Montréal)
-

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1. Flow of Infini members, PhD, guest post-doc, invited professors
2. Organization, scientific influence, contracts

4. Strategy

People

December, 2008

- Head : Alain Finkel (Prof. ENS Cachan)
- 3 Permanent members
 - Etienne Lozes (MCF) → Kassel
 - Sylvain Schmitz (MCF)
 - Philippe Schnoebelen (DR CNRS)
- 5 PhD students
 - Florent Bouchy (2006-2009) → Montréal, ing.
 - Rémi Brochenin (DGA/CNRS, 2006-2009) → post doc
 - Jules Villard (2007-2010) → London, post-doc
 - Jean-Loup Carré (EADS, 2007-2010) → Prof classes prépa
 - Pierre Chambart (2008-2011) → Univ. P7, ing.
- Delegations and post-doc
 - Peter Habermehl (MCF Paris7, sabbatical feb.2007-feb. 2009)
 - Pierre-Cyrille Héam (MCF Besançon, sabbatical 2008-2010)
 - Adam Antonik (post-doc)
- + tens of one-month invited professors: Ranko Lazic, Petr Jancar,...

June, 2013

- Head : Alain Finkel (Prof. ENS Cachan)
- 4 Permanent members
 - Stéphane Demri → New York
 - Laurent Doyen (CR) → Tempo
 - Sylvain Schmitz (MCF) → Dahu
 - Philippe Schnoebelen (DR CNRS)
- 5 PhD students
 - Mahsa Shirmohammadi (Doyen + Massart)
 - Amit Kumar Dahr (Demri + Sangnier)
 - Julien Reichert (Berwanger + Doyen)
 - Prateek Karandikar (Schnoebelen + Kumar)
 - Michael Blondin (F. + McKenzie)
- Chair and post-doc
 - Pierre Mckenzie (Digiteo chair)
 - Christoph Haase (post-doc)

9 PhD students

2008: Arnaud Sangnier (F.+ Lozes) CIFRE EDF

2009: Florent Bouchy (F.)

2009: Jean-Loup Carré (Goubault-Larrecq)

2010: Jules Villard (Demri + Lozes)

2010: Diego Figueira (Demri + Segoufin)

2011: Pierre Chambart (Schnoebelen)

2013: Rémi Brochenin (Demri+Lozes)

2012: Michael Cadilhac (F.+McKenzie)

January 2013: Rémi Bonnet (F.)

- 2010: Assistant-Prof. Université Paris 7
- 2013: Post-doc and engineer in Montréal
- 2013: Prof Math-sup
- 2013: Post-doc Londres
- 2013: Post-doc Edimbourg
- 2013: Engineer Université Paris 7
- 2013: Postdoc University of Genes
- 2013: Waiting for a post-doc
- 2013: Post-doc Oxford (Worrell, Ouaknine)

5 PhD in progress

Mahsa Shirmohammadi (Doyen + Massart)

Markov processes

Defense: 2014



Julien Reichert (Berwanger + Doyen)

Games on counters.

Defense: 2014



Amit Kumar Dahr (Demri + Sangnier)

Flat Counter Machines

Defense: 2015



Prateek Karandikar (Schnoebelen + Kumar)

Lossy Channel Systems & complexity

Defense: 2015

Michael Blondin (F. + McKenzie)

Reachability in Petri nets

Defense: 2016



Plan

1. Introduction

1. Scientific context
2. Forward reachability procedure
3. Backward reachability algorithm
4. Our two preferred models

2. WSTS and CM

1. Highlight: wqos and complexity
2. WSTS
3. Counter Machines

3. Management

1. Flow of Infini members, PhD, guest post-doc, invited professors
2. Organization, scientific influence, contracts

4. Strategy

Organization, scientific influence

- **Chairman**

GAMES 2011, PaVAS 2011, RP'2012, STSV 2012

- **Dagstuhl on infinite systems**

april 2014 (Esparza-F.-Mckenzie-Ouaknine)

- **Publications**

- 33 journal
- 71 conf

- **Collaborations**

- ∞ : papiers communs
- LSV\(∞): Goubault-Larrecq, Haddad,...
- Monde\LSV: Henzinger, Raskin, Jancar,...

- ANR Blanc « **Reachard** », LSV/LaBRI (2012-2014)

- Reachability in VASS and other models
- <http://www.lsv.ens-cachan.fr/Projects/anr-reachard/>

Etienne Lozes

(Kassel, Allemagne)
HDR: 3 july 2012



Laurent Doyen

HDR: 13 march 2012



- ANR « DYNRES » (2011-2013)

- logics for ressources
- <http://anr-dynres.loria.fr/>

- + 8 contracts (finished)

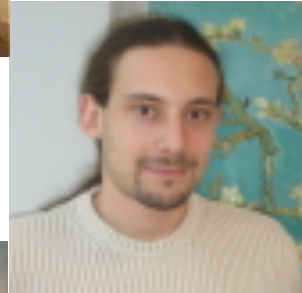
December, 2nd, 2013

1. **Stéphane Demri**
Dahu - infini
Marie Curie New York (2012 - 2014)



2. **Alain Finkel**

3. **Sylvain Schmitz**
MCF (2008 -->)
--> Dahu INRIA (2013-2015)



4. **Philippe Schnoebelen**

5. **Laurent Doyen**
CR CNRS (2009 -->)
--> Tempo (sept'13 -->)



6. **Pierre McKenzie**
Univ. Montréal
Chaire DIGITEO ENS Cachan-Ecole X
2013 – 2014



Plan

1. Introduction

2. WSTS and CM

1. Highlight: wqos and complexity.
2. WSTS
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3. Management

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Check the work of Infini...

Did Infini realize its 2008 objectives ?

From 2008 ... to 2013

Perspectives 2008-2013

(AERES, dec. 2008)

1. Verification of heterogeneous systems : CM,...

2. Verification of systems with dynamic memory

3. Verification of WSTS

Assessment 2008-2013

(june 2013)

1. CM

2.

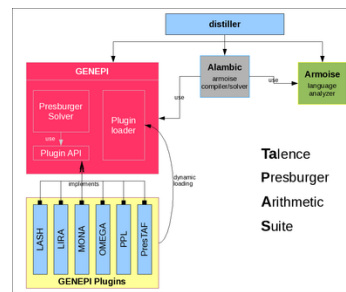


3. WSTS

Report of AERES (2008) versus our balance (2013)

Report AERES 2009

1. Strengths:
 - Publications, CM
 - **FAST**
2. To improve
 1. Interaction with program analysis
 2. + cooperation LIAFA
 3. **Data dyn.**
3. Recommendations
 1. Recrut a CR
(prog Analysis-dem. Aut.)
 1. **Develop FAST**



Bilan 2008-2013

1. Strengths :
 - publications, CM
 - WSTS
2. What has been done
 1. More theory WSTS+CM
 2. + cooperation LIAFA
(Habermehl, PhD Dahr)
3. What has been done
 1. 2009: Recruitment Laurent Doyen, CR CNRS, on Games
 2. **FAST stopped but TAPAS continue**
<http://tapas.labri.fr/trac>



Objectives and challenges

2013 --> 2017



- **Constat**

- We have formed brilliant researchers in and out of the LSV
- We inspired fruitful research in France and outside (Oxford, Bruxelles, Vienne, Chennai, Madrid).

- **Objectives**

- Strengthen our leadership role in WSTS + CM
- Recruit a colleague on verification algorithmics
- Recruit PhD students



- **Challenges**

- Classify WSTS
- Complete the theory of complete WSTS
- Move towards algorithmics (WSTS + CM)

The end

Questions...