Efficient Approximations of Conjunctive Queries

Pablo Barceló (University of Chile)
Leonid Libkin (University of Edinburgh)
Miguel Romero (University of Chile)

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- APPROXIMATE!

Approximation idea, by picture

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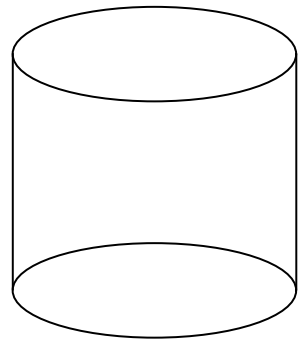

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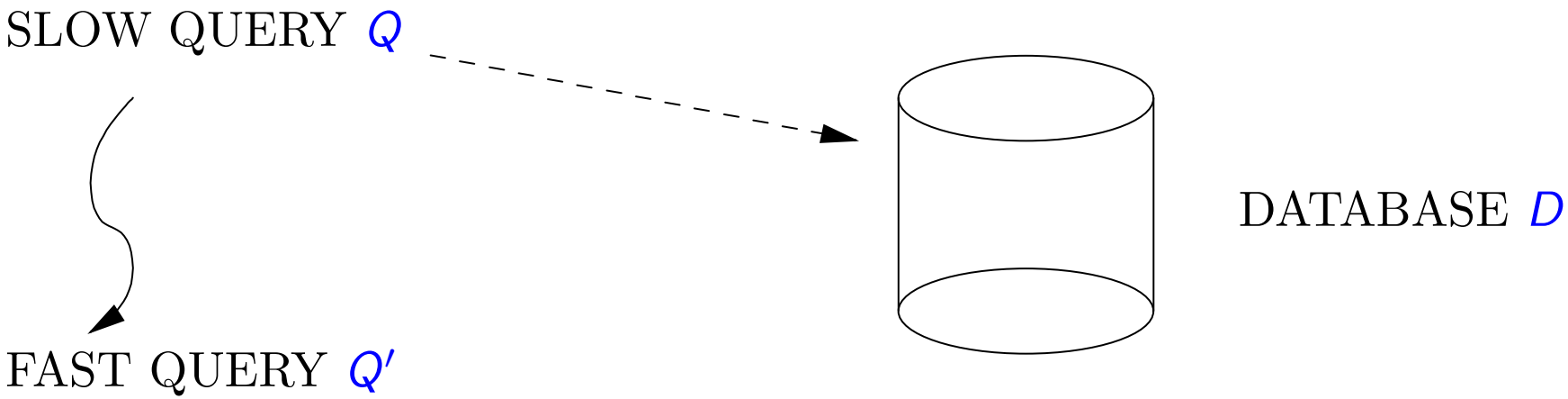

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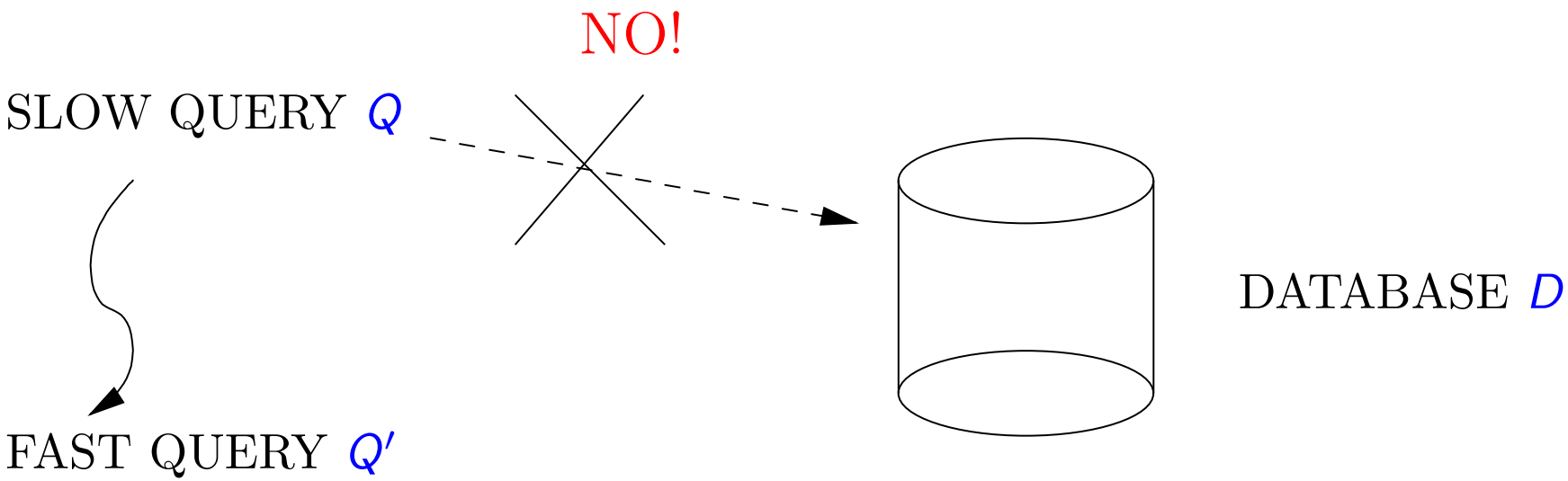

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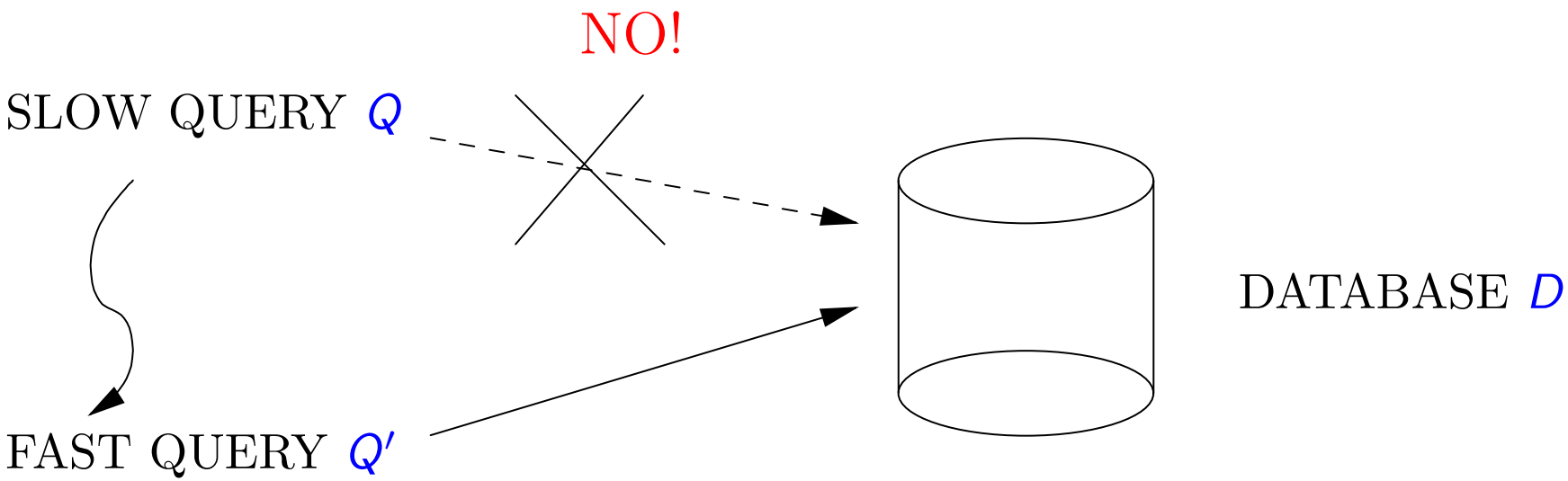

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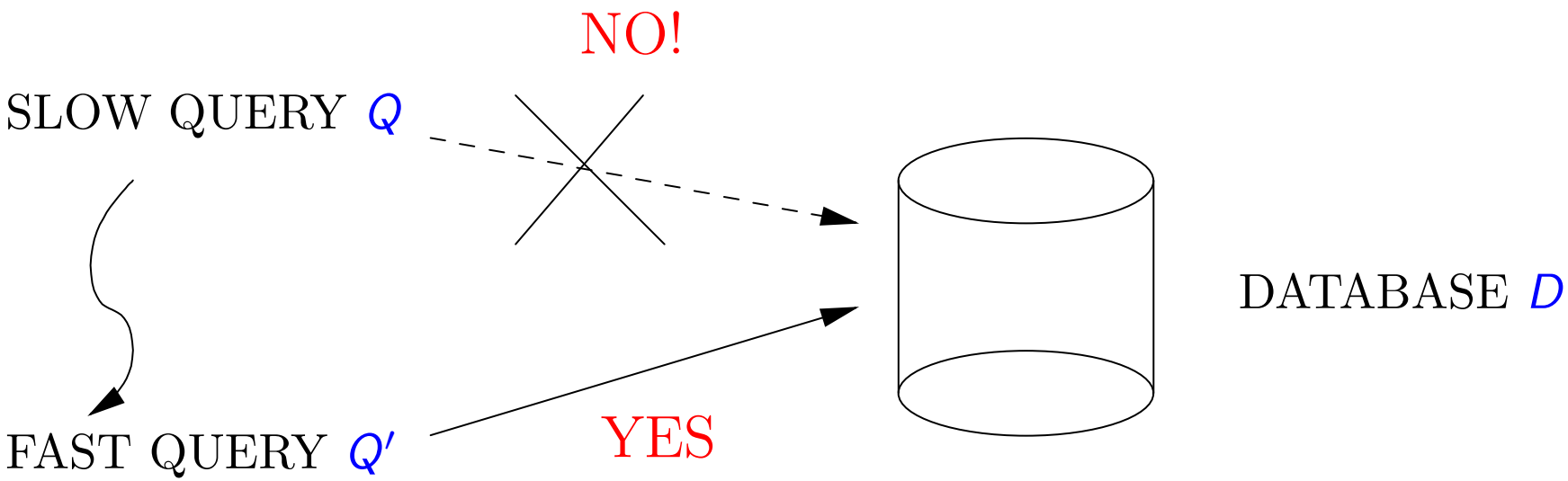

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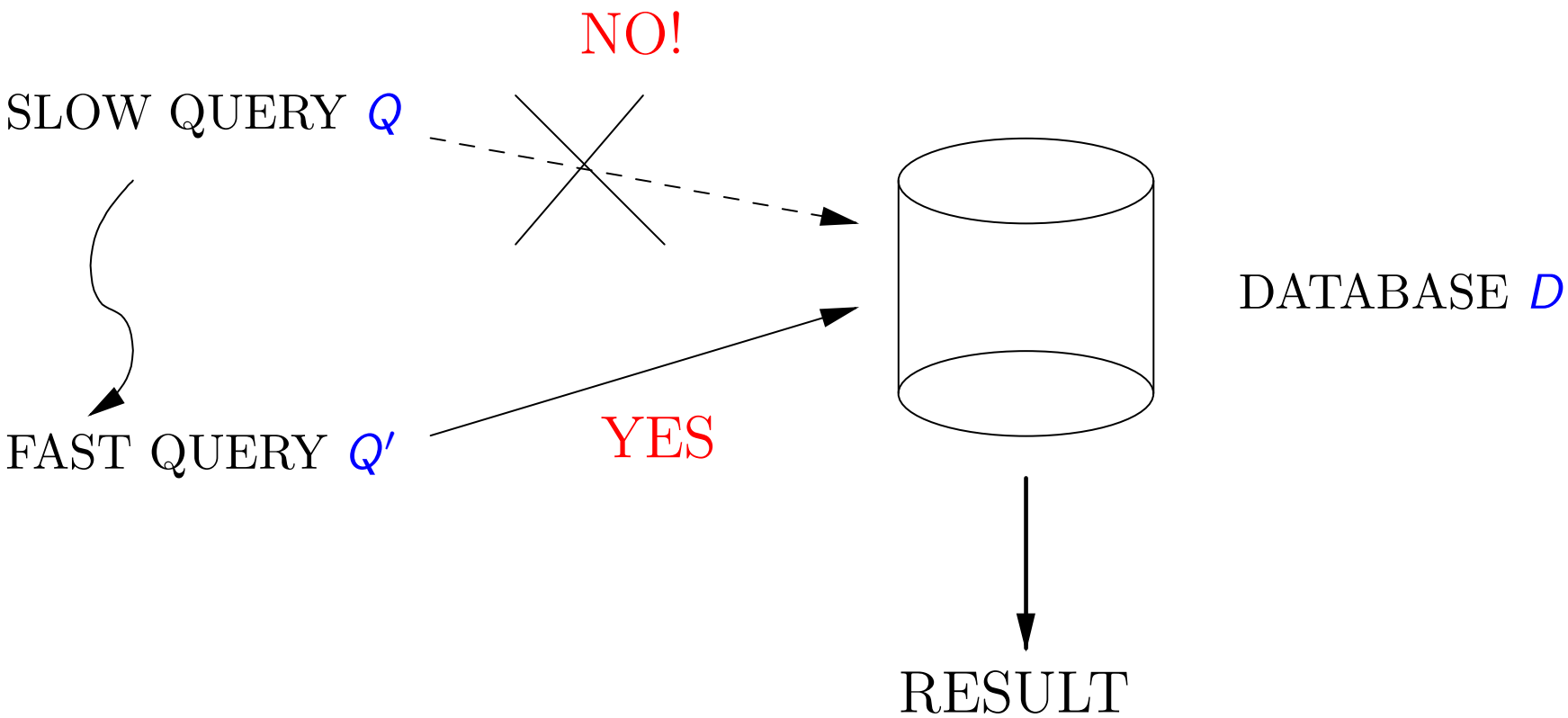

Approximation techniques

The idea is, of course, quite old.

Standard approaches include:

- Statistical methods, e.g. histograms, combined with data mining techniques (Ioannidis ICDT 2003 survey + many others).
  - uses both data and query
- Semantic relaxations based on traditional data management techniques (Fan ICDT 2012 survey).
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Our choice: static analysis.
Once a query is approximated, it can be run and re-run when updates are applied.

Desiderata for approximations

- They must be fast
- They must be close to the queries they approximate

Next: formulate these two requirements for conjunctive queries.

Conjunctive queries (CQs)

$$\underbrace{Q(\bar{x})}_{\textit{head}} \quad :- \quad \underbrace{S_1(\bar{u}_1), \ldots, S_n(\bar{u}_n)}_{\textit{body}}$$

- atoms $S_i(\bar{u}_i)$: each S_i is a relation symbol, each $\bar{u}_i$ a list of variables.
- variables $\bar{y}$ in the body but not in the head are existentially quantified:
  $Q(\bar{x}) \;=\; \exists \bar{y}\, \big(S_1(\bar{u}_1) \wedge \cdots \wedge S_n(\bar{u}_n)\big)$.

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- Why CQs?
  - a very important and common class of queries
  - equivalent to select-project-join queries
  - we know a lot about their evaluation.

Tableaux, evaluation, containment

$$Q(\bar{x}) \quad :- \quad S_1(\bar{u}_1), \ldots S_n(\bar{u}_n)$$

Its tableaux is the body of Q viewed as a database:

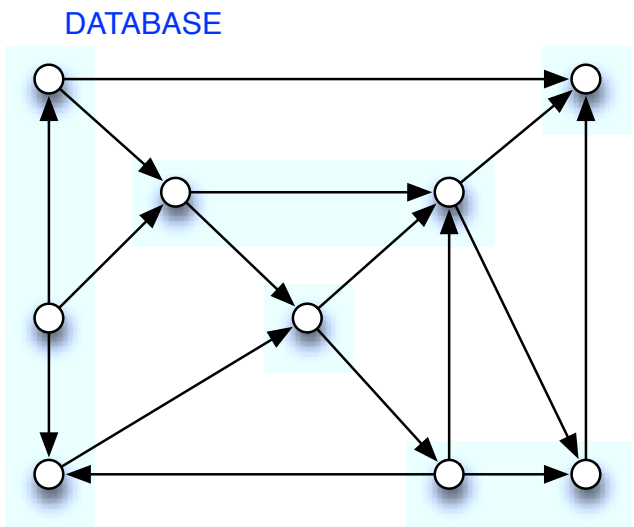
$$T_Q \quad = \quad (\{S_1(\bar{u}_1), \ldots, S_n(\bar{u}_n)\}, \quad \bar{x})$$

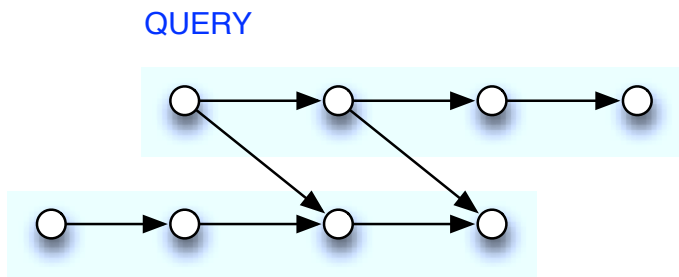
Evaluation and static analysis via tableaux and homomorphisms:

- $\bar{a} \in Q(D) \Leftrightarrow$ there is a homomorphism $T_Q \to (D, \bar{a})$
- $Q \subseteq Q' \Leftrightarrow$ there is a homomorphism $T_{Q'} \to T_Q$

CQ evaluation, graphically

Goal: Find a homomorphism from T_Q into D.





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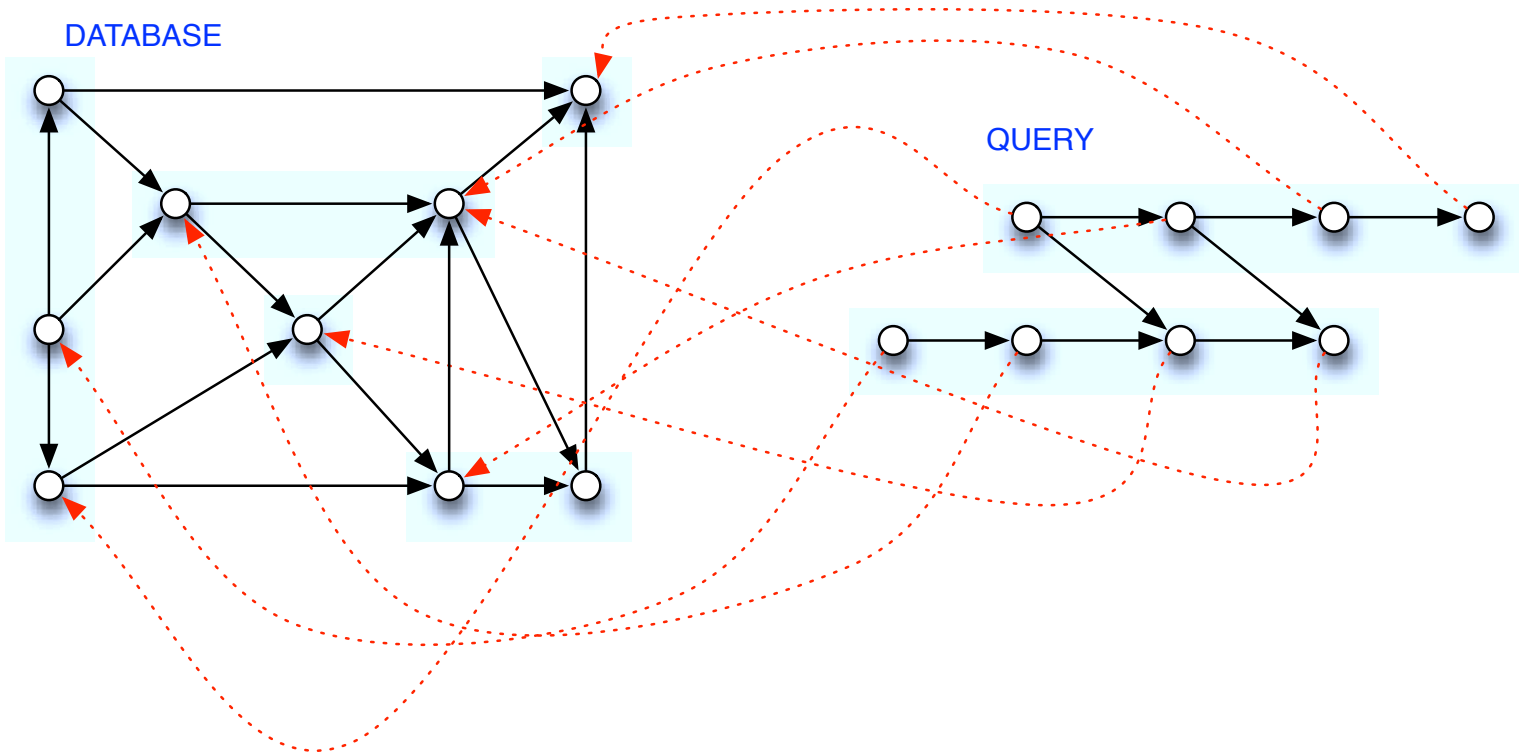
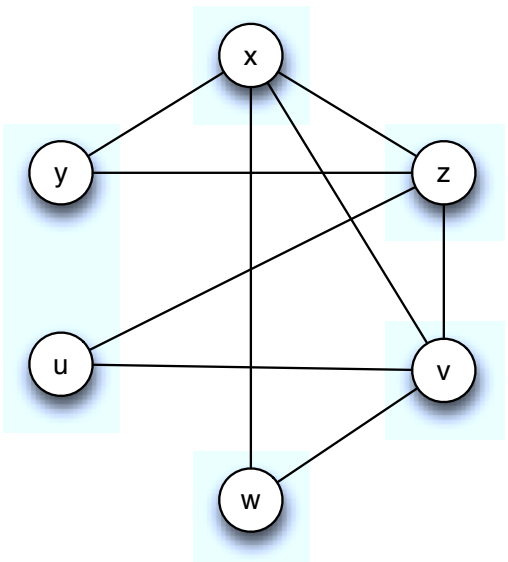


Good classes of CQs

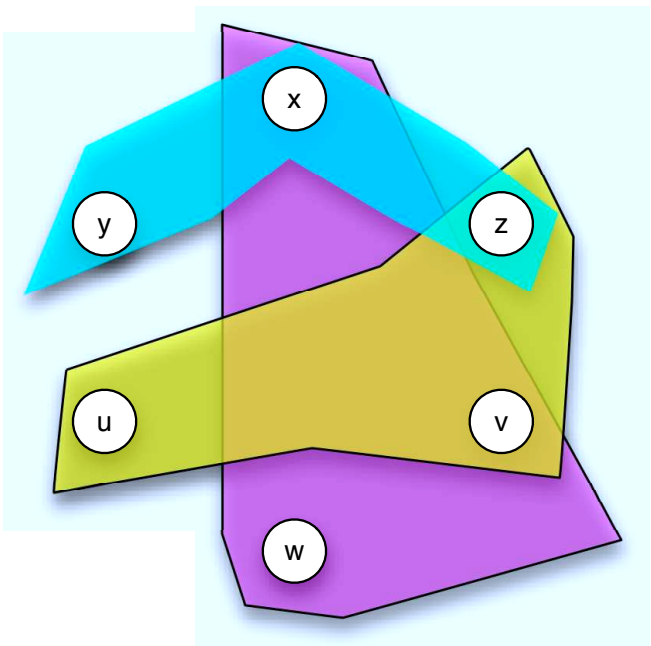
Two kinds: graph-based and hypergraph-based

$$Q \;:-\; R(x,y,z), R(z,u,v), R(v,w,x)$$

graph of the query, $G(Q)$



hypergraph of the query, $H(Q)$



Good classes of CQs – acyclicity and relatives

A general idea of tractable restrictions for CQs: acyclicity
(Yannakakis 1981 – linear-time evaluation)

Extensions:

- for graph-based notions: bounded treewidth
  - (Chekuri, Rajaraman, Kolaitis, Vardi, Grohe, Flum, Segoufin, Schwentick)
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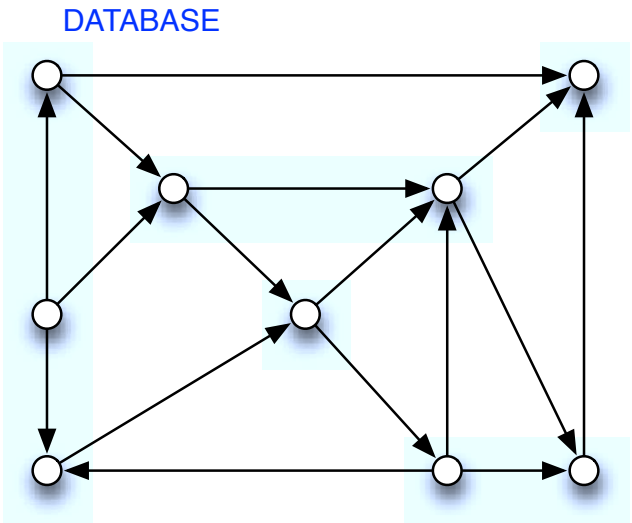
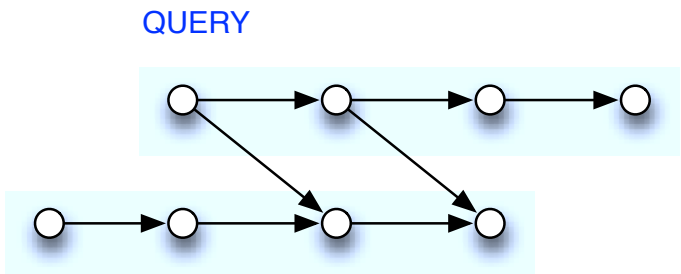
Revised goal: approximate within tractable classes of CQs

Approximating CQs, graphically

Given Q and D





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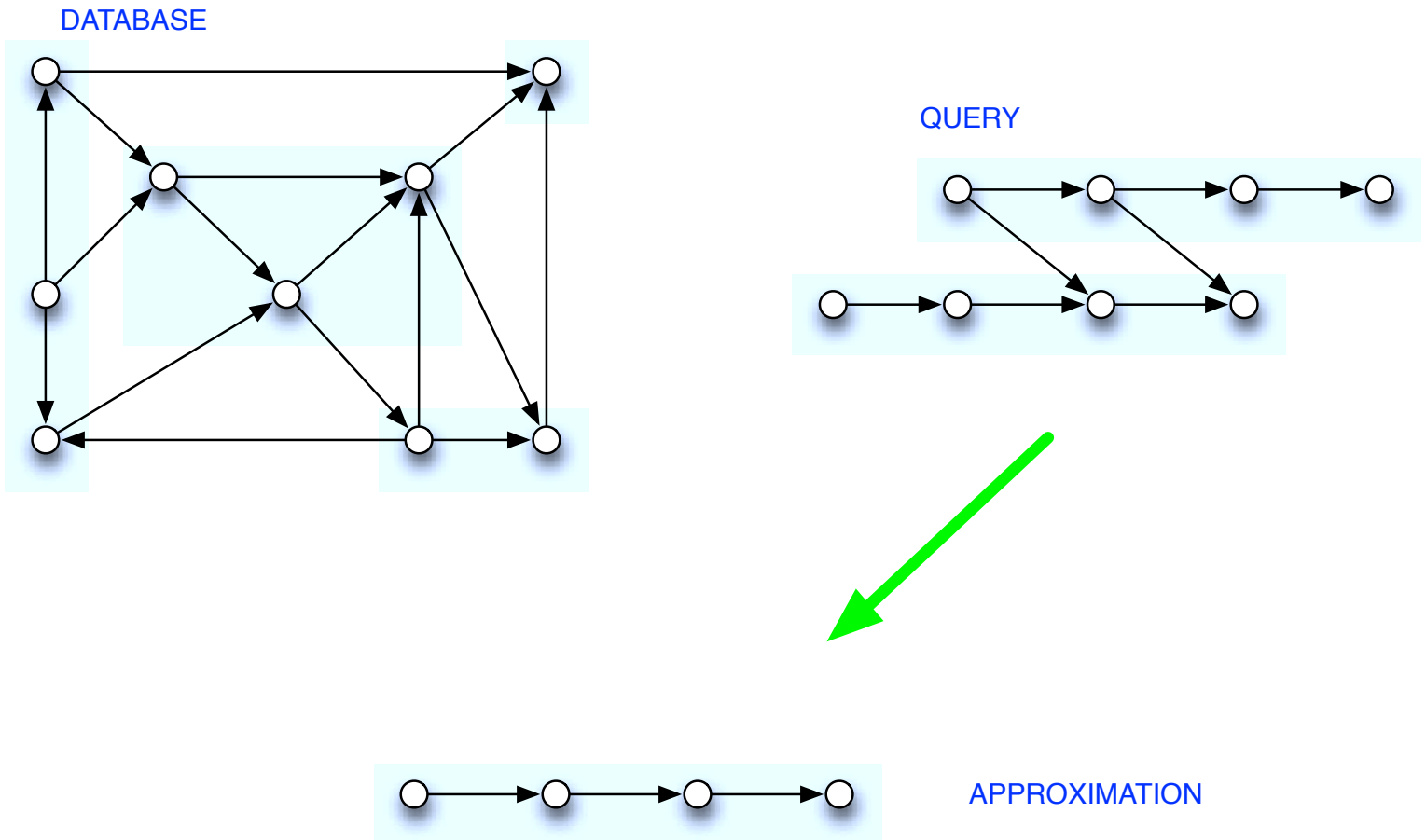
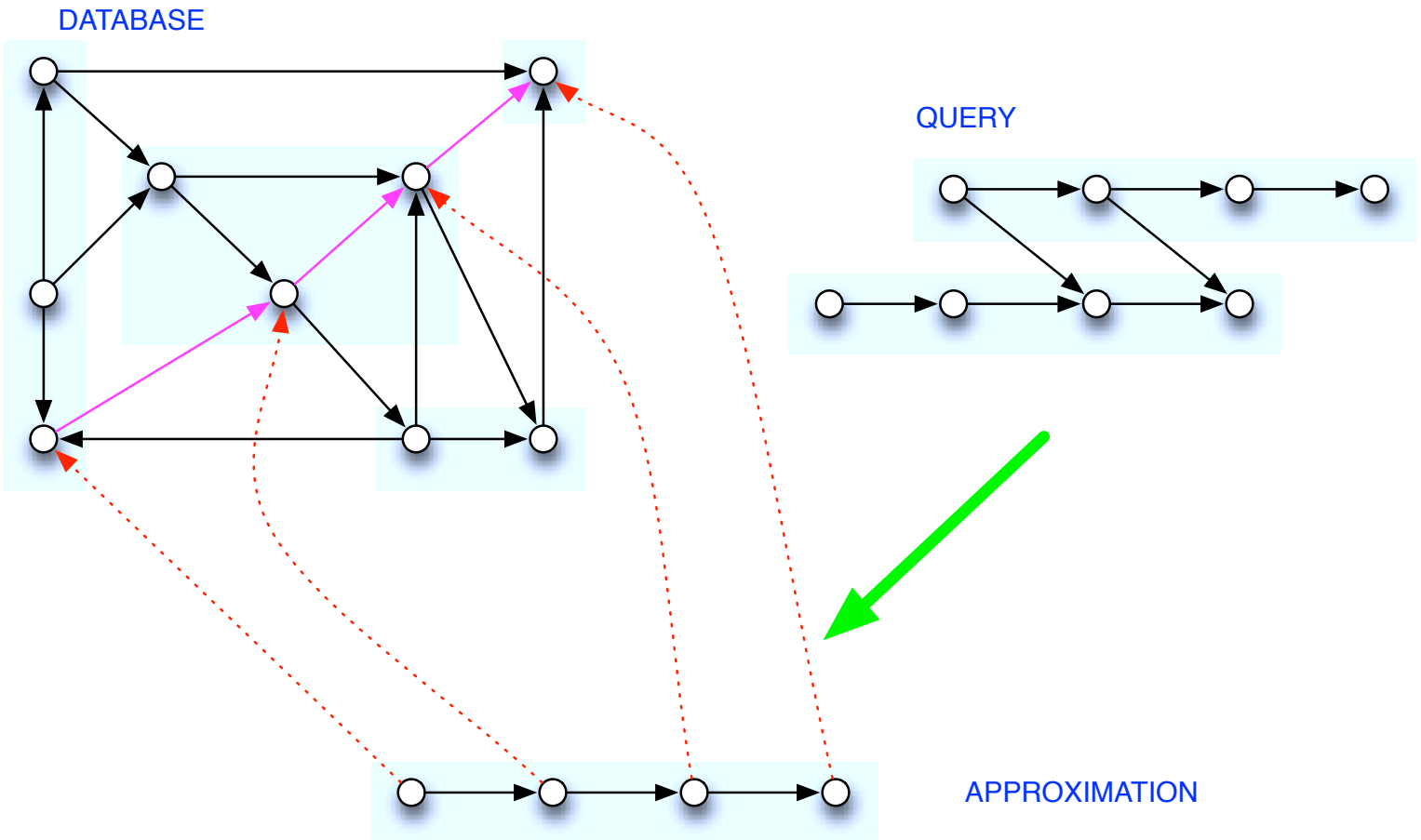
Given Q and D, find a tractable approximation



Approximating CQs, graphically

Given Q and D, find a tractable approximation and evaluate on D



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  - hard to hope for more given the complexity of static analysis for CQs
- For example, using acyclic approximations we have complexity $2^{O(|Q|\cdot\log|Q|)} + |D| \cdot |Q|^k$
- Much faster than $|D|^{O(|Q|)}$ on large databases.

CQ approximations: Definition

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Definition A query $Q' \in \mathcal{C}$ is a $\mathcal{C}$-approximation of Q if:

- $Q' \subseteq Q$; and
- there is no $Q'' \in \mathcal{C}$ such that $Q' \sqsubset_Q Q''$.

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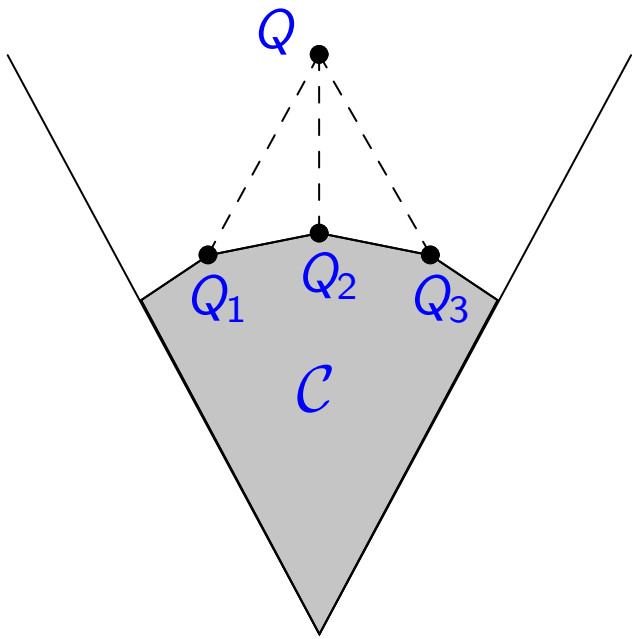
$$Q' \sqsubset_Q Q'' \quad \Leftrightarrow \quad Q' \sqsubseteq_Q Q'' \text{ and } Q'' \not\sqsubseteq_Q Q'.$$

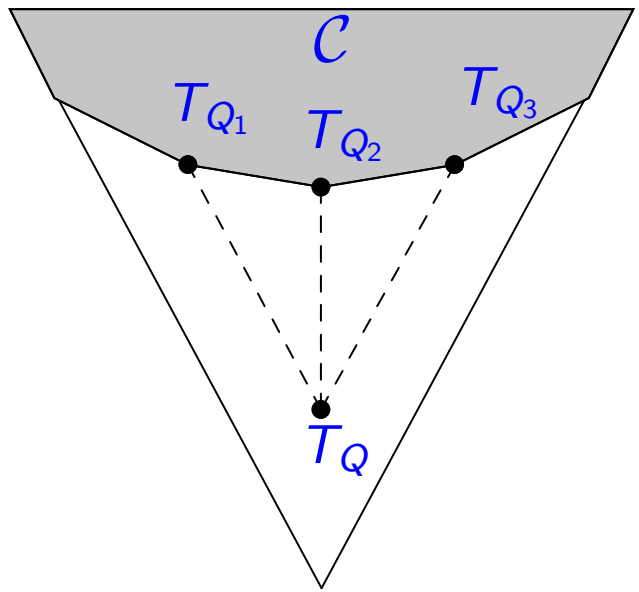
CQ approximations, graphically

Proposition

We can use the familiar containment/homomomorphism (pre)ordering instead of $\sqsubset$*:*



a query view:
based on containment



a tableau view:
based on homomorphism

Existence of approximations: graph-based case

A class $\mathcal{C}$ of queries is closed under substructures if:

$$Q \in \mathcal{C} \text{ and } T_{Q'} \subseteq T_Q \implies Q' \in \mathcal{C}$$

Example: classes of queries of bounded treewidth.

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Example: classes of queries of bounded treewidth.

A small assumption: the trivial query $Q :- R(x, x, \ldots, x)$ is in $\mathcal{C}$.

Does not hurt at all:

- this query is of treewidth 1 and acyclic, i.e. belongs to all tractable classes;
- it is also contained in every other CQ.

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- *An approximation can be found in single-exponential time*
  - $2^{O(n \log n)}$ *to be precise*
- *There are at most exponentially many non-equivalent approximations.*

Example

- Take

$$Q :- R(x, y, z),\ R(y, x, u),\ R(u, z, x)$$

- Its treewidth is 3 – maximum possible for a query with 4 variables; its graph is K_4.
- Has approximations of the smallest possible treewidth, i.e., 1:

$$Q' :- R(x, y, y),\ R(y, x, y),\ R(y, y, x).$$

- Good to reduce treewidth as it determines the exponent in query evaluation.

Bounds on static analysis

In general, exponential bounds are unavoidable for analysis of CQs (the most basic containment problem is NP-complete).

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- *the size of* Q_n *grows linearly in* n*, and*
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Q_ns can be Boolean CQs on graphs, so the bound applies to acyclic approximations as well.

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Input:	A CQ Q and another CQ $Q' \in \mathcal{C}$.
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- Hardness proven for Boolean CQs on graphs.
- Nontrivial reduction from the problem whether a graph is 4- but not 3-colorable (Riege, Rothe, 2006).

Hypergraph-based classes: acyclicity

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Example: Decomposition for the query
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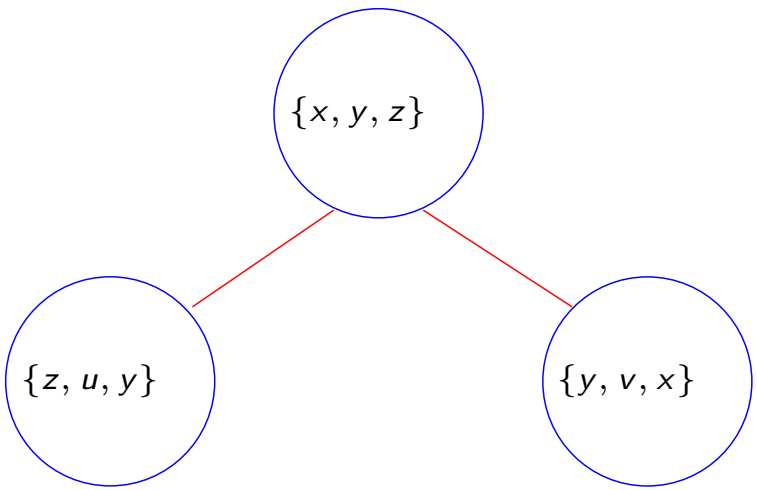

Acyclicity and hypertreewidth

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If Q is an acyclic conjunctive query, then checking whether $\bar{a} \in Q(D)$ can be done in time $O(|D| \cdot |Q|)$.

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Generalization: hypertree width (Gottlob, Leone, Scarcello, 2000).

- Extends the notion of treewidth.
- Acyclicity = hypertree width 1.
- Guaranteed tractable evaluation of CQs:
  - Hypertree width determines the exponent.
  - $O(|D|^c \cdot p(|Q|))$.

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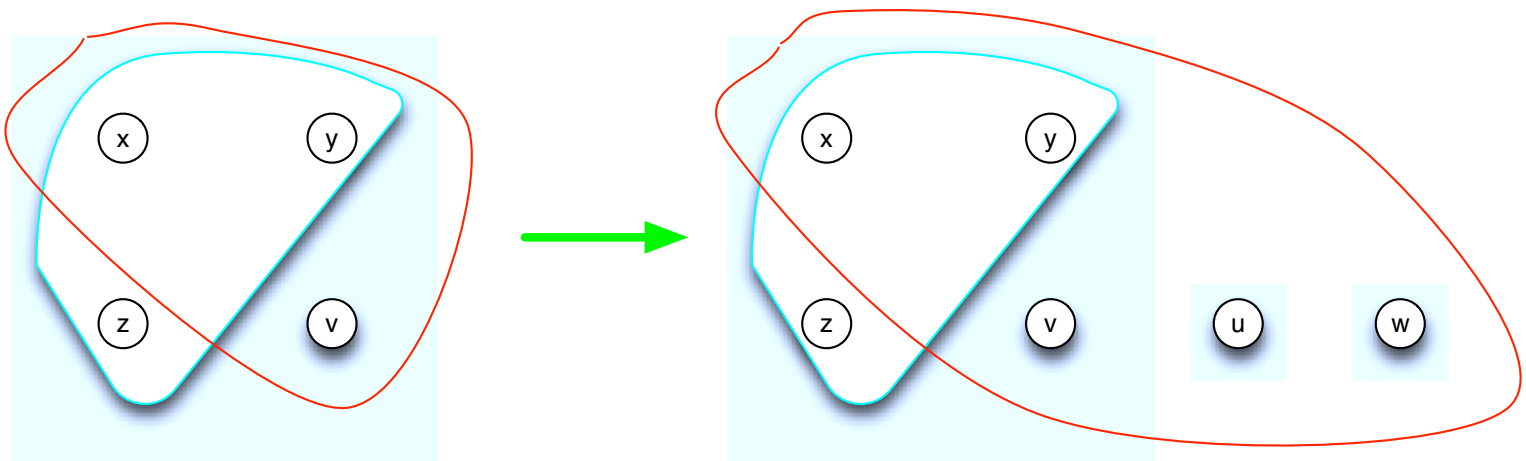
Instead, we use two new closure conditions:

1. Closure under induced sub-hypergraphs;
2. Closure under edge extensions:



New closure conditions and hypertreewidth

The closure conditions are good for us:

Proposition

The class of hypergraphs of hypertree width $\leq k$, for every $k \geq 1$, is closed under both induced subhypergraphs and hyperedge extensions.

In particular, the class of acyclic hypergraphs satisfies these closure conditions.

Existence of approximations for hypergraph-based classes

$\mathcal{C}$ – a class of CQs whose hypergraphs are closed under induced subhypergraphs and edge extensions:

- for example, queries of hypertree width $\leq k$, or
- acyclic queries.

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All lower bounds shown for graphs, hence apply here.

Example of acyclic approximations

A cyclic query:

$$Q \text{ :- } R(x,y,z),\ R(z,u,v),\ R(v,w,x)$$

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$$Q_3 \ :- \ R(x,y,x)$$

Future work

- Quantitative analysis
  - no fake results like: if Q' is a $\mathcal{C}$-approximation of Q, then with probability 1 they agree on a randomly chosen database (under the uniform distribution)
- More relaxed approximations: do not insist on $Q' \subseteq Q$
- Analyze generalized hypertree width as a means of getting good approximations.