Guarded Negation

Balder ten Cate

based on joint work with Luc Segoufin, Vince Barany and Martin Otto
(STACS 11, ICALP 11, VLDB 2012)

Theme

- **Theme**: decidable fragments by restricting the use of negation.
  - Unary negation: allow only $\neg\phi(x)$ [tC & Segoufin STACS 11]
  - Guarded negation: allow also $G(\mathbf{x}) \wedge \neg\phi(\mathbf{x})$ [Barany, tC & Segoufin ICALP 11]

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- Orthogonal to previous ways of "taming" FO:
  - We make no restriction on the number of variables
  - We make no restriction on quantifier alternation.
  - We allow unguarded quantification.

Modal logic

- (Basic) modal logic: a small fragment of FO
  - Signature: $\{R, P_1, \ldots, P_n\}$
  - Formulas: $\phi(x) := P_i(x) \mid \phi(x) \wedge \psi(x) \mid \neg\phi(x) \mid \exists y(Rxy \wedge \phi(y))$

shorthand: $\Diamond\phi$

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- Very well behaved (decidable for satisfiability, has Craig interpolation, ..)
  - many extensions, such as the modal mu-calculus, are decidable too.
- "Why is modal logic so robustly decidable?" (Vardi '96)
  - tree model property,
  - translation into MSO (tree automata),
  - finite model property.

Why Modal Logic is so Robustly Decidable

- "Syntactic explanations":
  - Modal formulas only need two variables (FO^2) [Graedel-Kolaitis-Vardi 1997]
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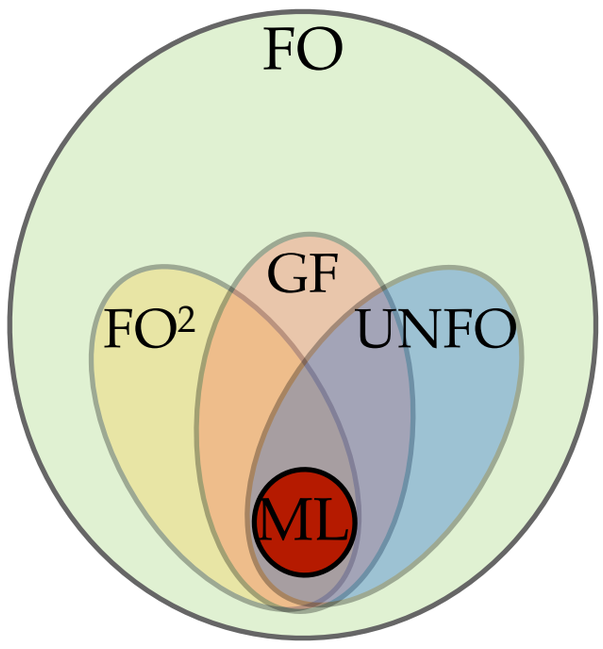

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  - $\phi ::= R(\mathbf{x}) \mid x = y \mid \phi \wedge \phi \mid \phi \vee \phi \mid \exists x \phi \mid \neg\phi(x)$
  - Only allow negation of formulas in one free variable.
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- Fixed-Point Extension (UNFP):
  - $\phi ::= \ldots \mid [\mathrm{LFP}_{Z,z}\, \phi(Z,\mathbf{Y},z)](x)$ (where Z occurs only positively in ϕ)
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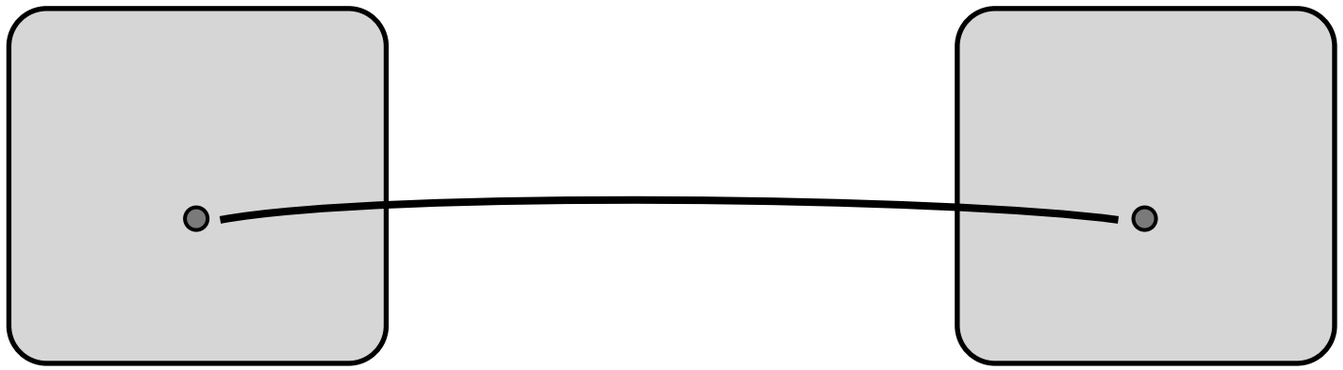
- UNFO and UNFP generalizes many existing logics:
  - Modal logic, modal mu-calculus, various description logics,
  - Unions of conjunctive queries, monadic Datalog,
  - CTL*(X), Core XPath

UNFO and UNFP

- Good model theory:
  - UNFO & UNFP have the tree-like model property (bounded tree-width)
  - UNFO has the finite model property
  - UNFO has Craig interpolation
  - UNFO can be characterized in terms of UN-bisimulation invariance.
- Decidable for satisfiability
  - 2ExpTime-complete, both on arbitrary structures and in the finite (via [Bojanczyk '02])
- Model checking:
  - UNFO model checking: $P^{NP[\log^2]}$ - complete (via [Schnoebelen '03])
  - UNFP model checking: in $NP^{NP} \cap coNP^{NP}$ and P^{NP}-hard.

UN-bisimulation game



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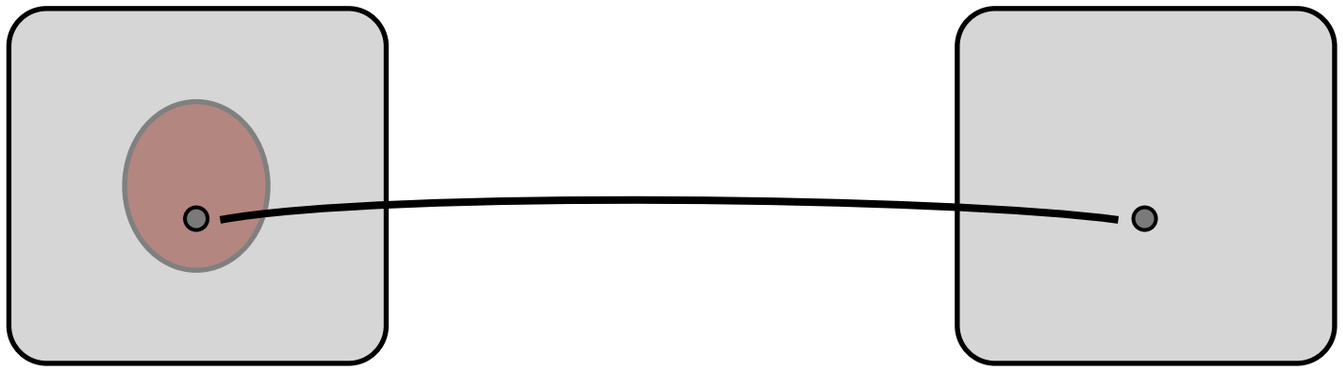

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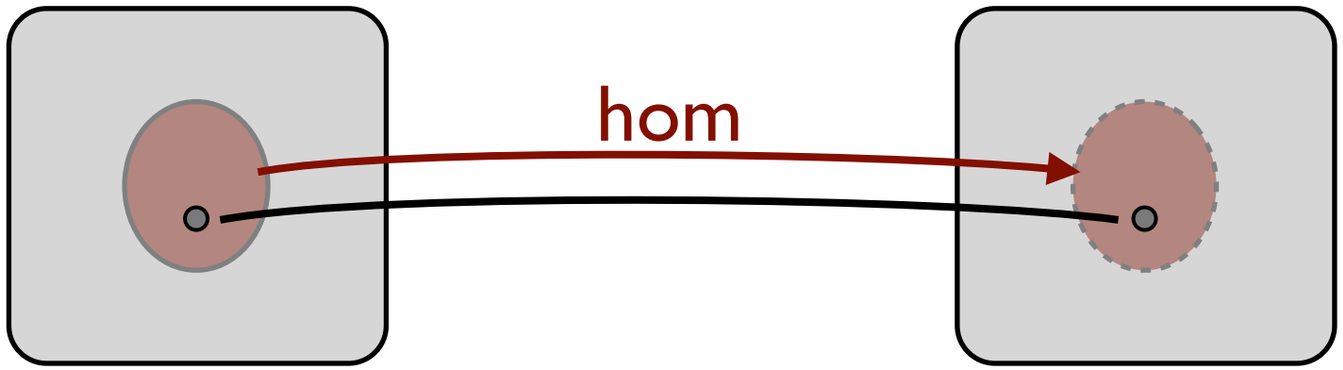

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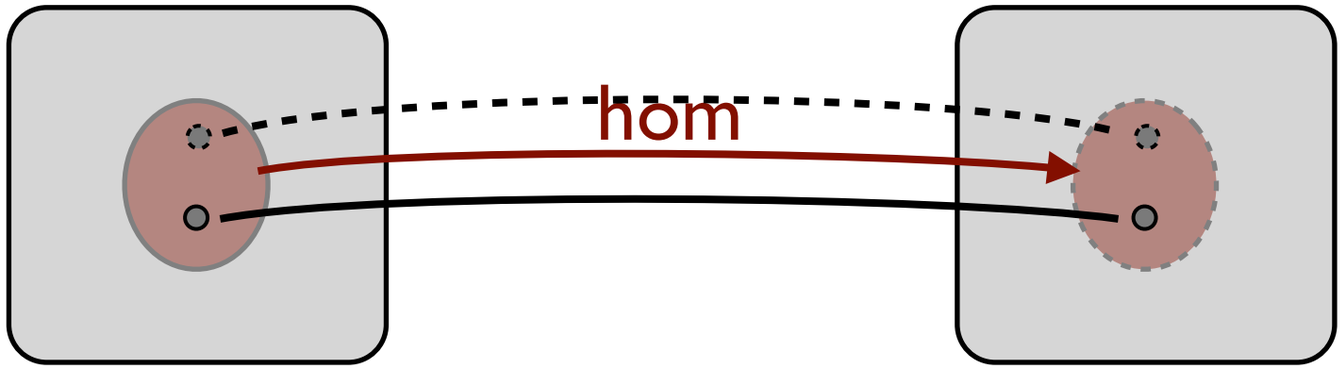

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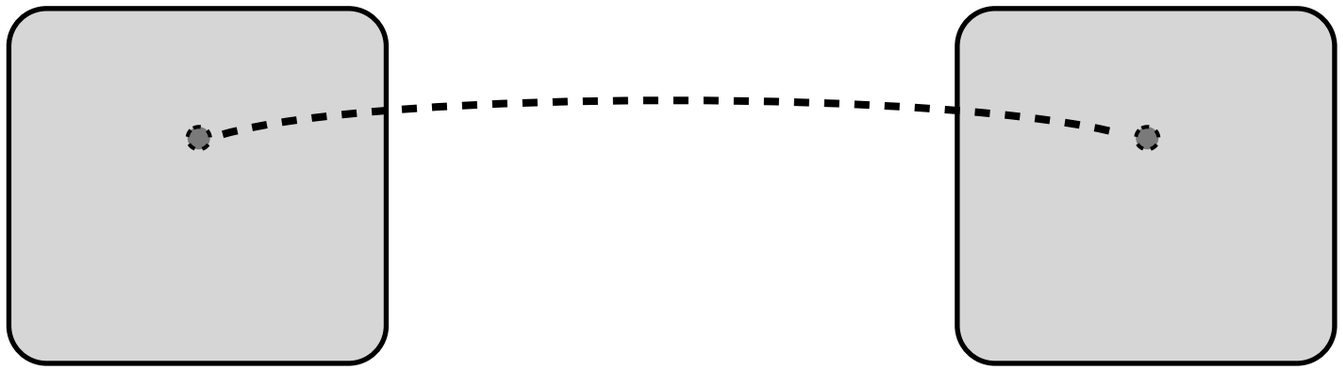

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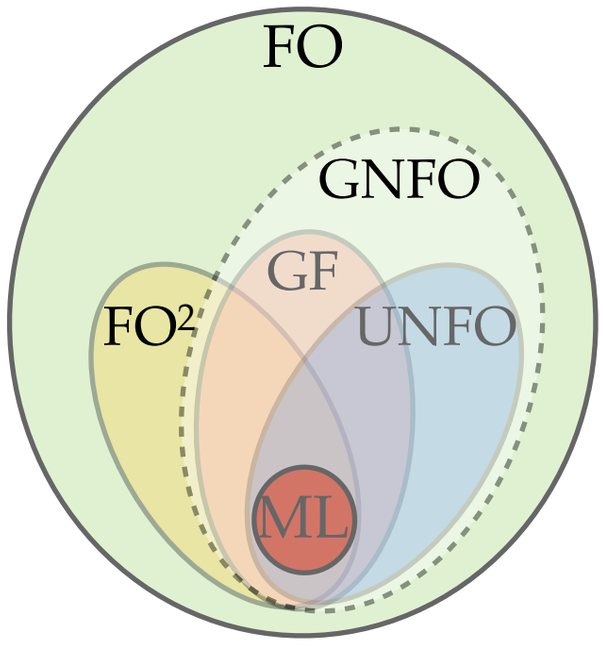

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  - (greatest fixed points can be expressed as dual)
- **Fact**: Every GFO / GFP sentence is equivalent to a GNFO / GNFP sentence.

Normal form

- GN-Normal form for GNFO formulas:

  $q[\phi_1/U_1, ..., \phi_n/U_n]$
  (where q is a UCQ containing relations U1, ..., Un)

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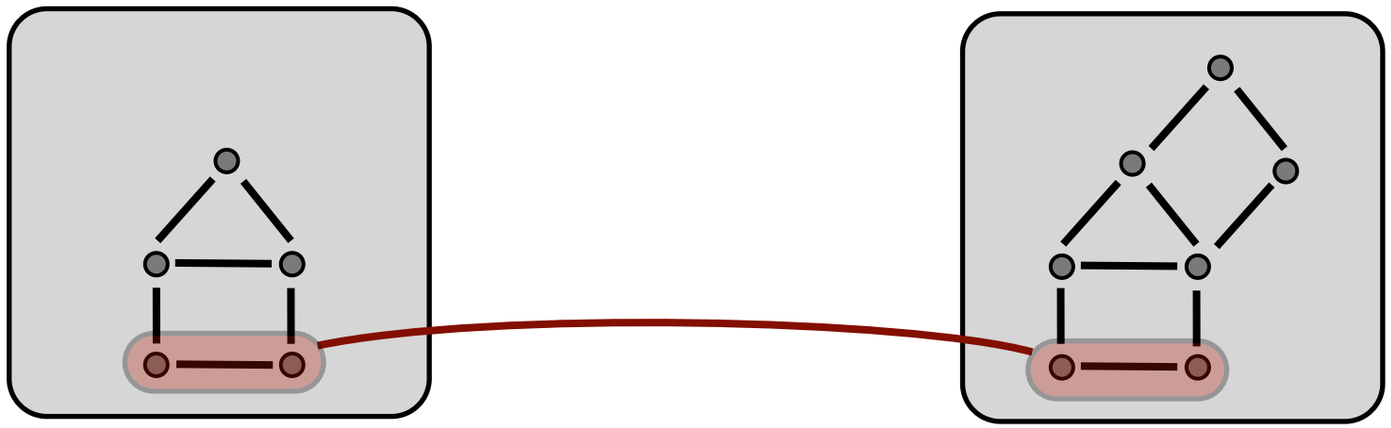
- **Fact**: Every GNFO / GNFP formula is equivalent to one in GN-normal form.

  - GNFO^k / GNFP^k: as above, but only use queries q with at most k variables.
  - GFO / GFP: as above, but only allow acyclic queries.

GN-Bisimulation Game



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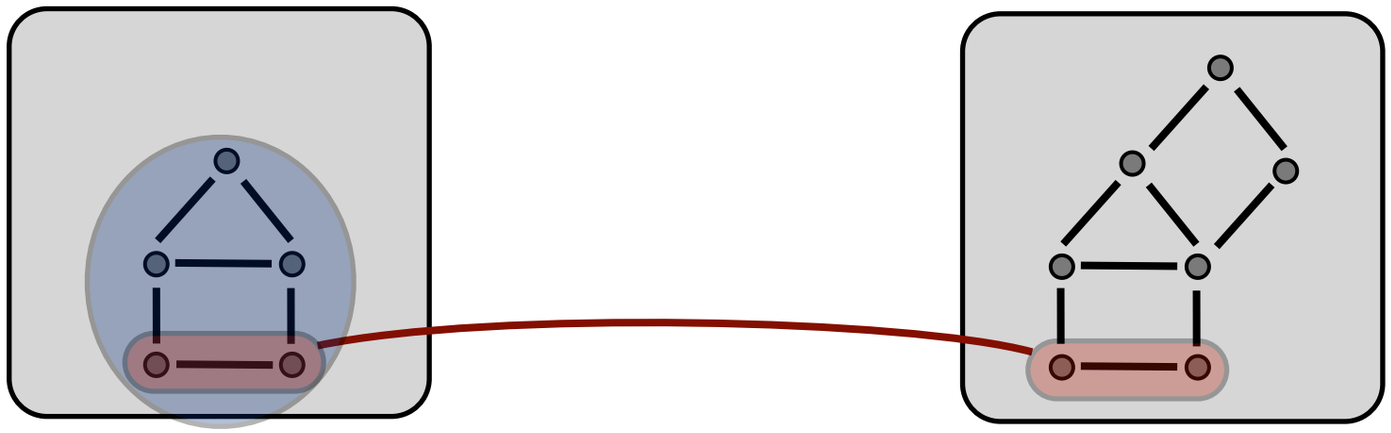

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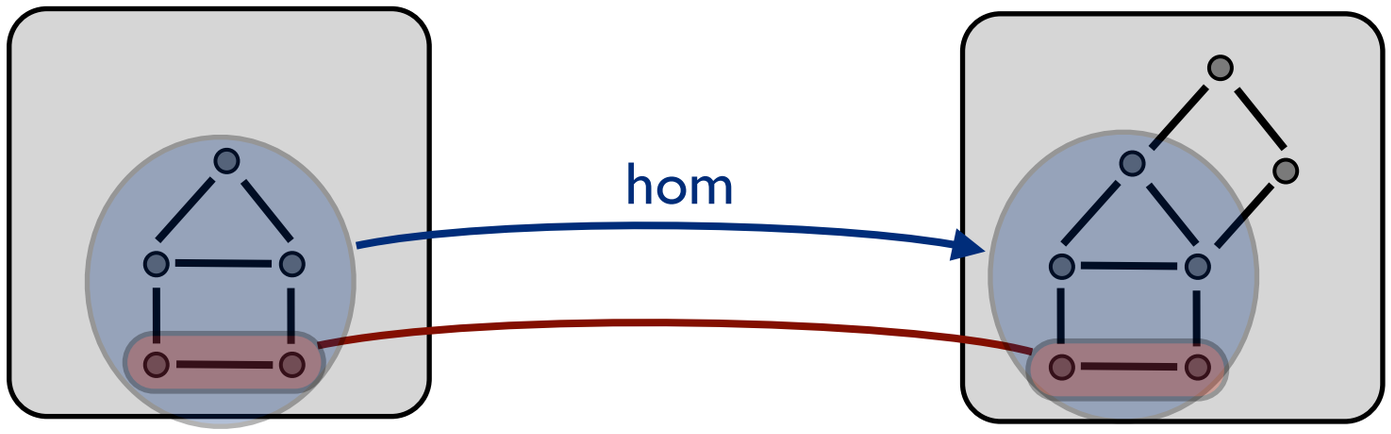

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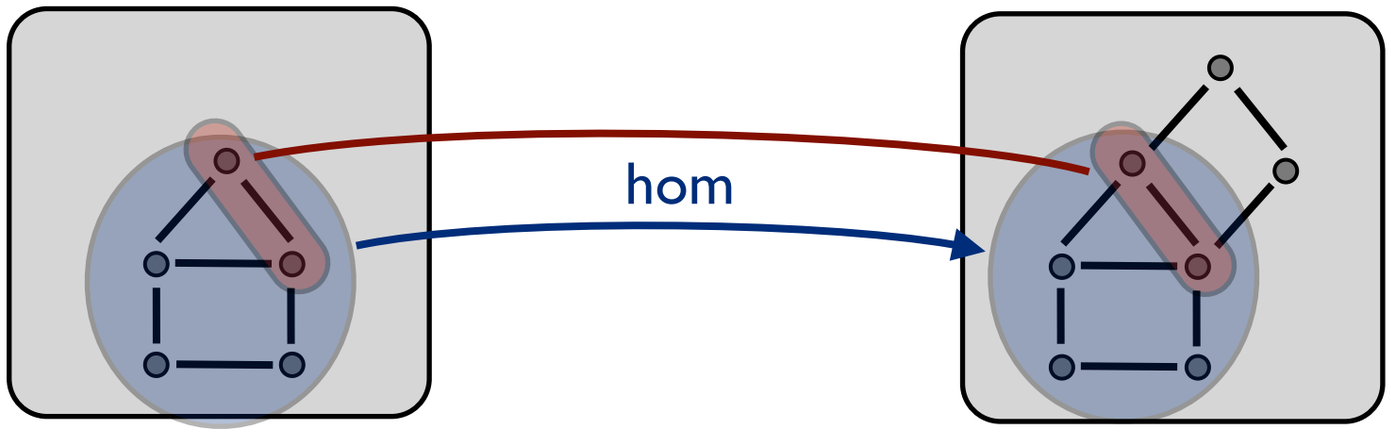

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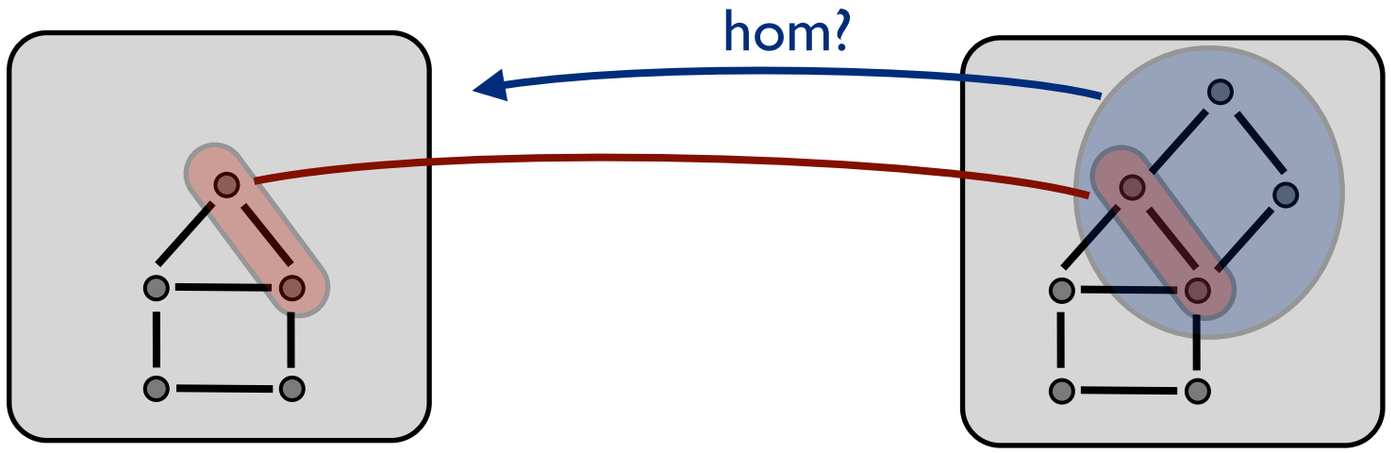

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GNFO and GNFP

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  - GNFO & GNFP have the tree-like model property (bounded tree-width)
  - GNFO has the finite model property (using [Barany-Gottlob-Otto 2010])
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- Decidable for satisfiability
  - 2ExpTime-complete, both on arbitrary structures and in the finite (using [Barany-Gottlob-Otto 2010; Barany-Bojanczyk 2011])
- Model checking:
  - GNFO: $P^{NP[\log^2]}$ - complete. GNFP: in $NP^{NP} \cap coNP^{NP}$ and P^{NP}-hard.
- Decidable boundedness problem for GNFO (using [Colcombet-Loeding; Blumensath-Otto-Weyer])

Three Examples

- Three “applications” of guarded negation.

1: Querying the Guarded Fragment

- Barany-Gottlob-Otto 2010 ("Querying the guarded fragment"):
  - The following is 2ExpTime-complete and finitely controllable:

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    - Replace all UCQs in the formula by fresh predicates, and "axiomatize" them (a la Scott normal form) using GFO sentences and negated CQs.
  - We show GNFP satisfiability is 2ExpTime-complete using the techniques from [Barany-Gottlob-Otto 2010] as well as [Barany-Bojanczyk 2011].

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- **Open-world query answering**: given an (incomplete) database D, a set of constraints T, and a (Boolean) query, is it the case that $D \wedge T \models q$?

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  - NP-complete for data complexity (we identified a large PTime fragment).

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  - Rules of the form $R(\mathbf{x}) :- (\neg)R_1(\mathbf{x_1}), ..., (\neg)R_n(\mathbf{x_n})$
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- GN-Datalog contains Monadic Datalog and Datalog LIT.
- **GN-Datalog is contained in GNFP.** Therefore, containment is decidable (2ExpTime-cmp). Boundedness (suitably defined) is decidable as well.

Decidability of GNFP Satisfiability

- **Thm**: (Finite) satisfiability of GNFP is 2ExpTime complete.
- Main ingredients of the proof:
  - Treeifications of cyclic conjunctive queries,
  - Rosati covers ([Barany-Gottlob-Otto 2010])
  - A reduction from GNFP to GFP.

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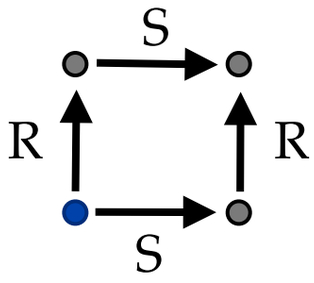

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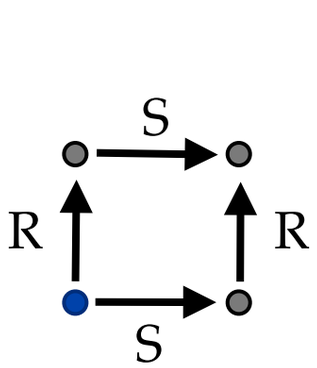

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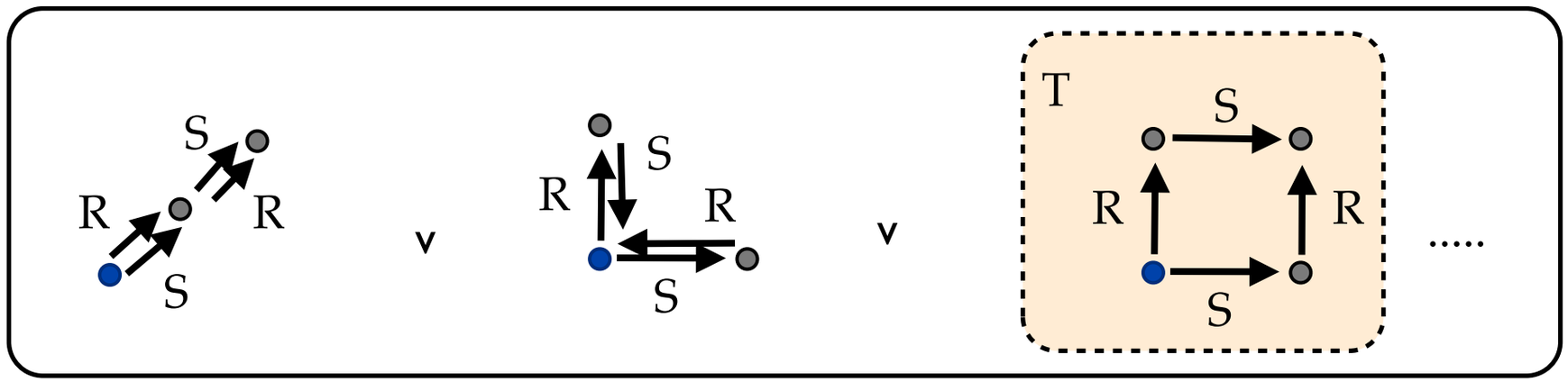

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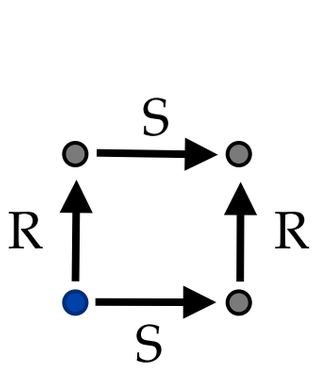

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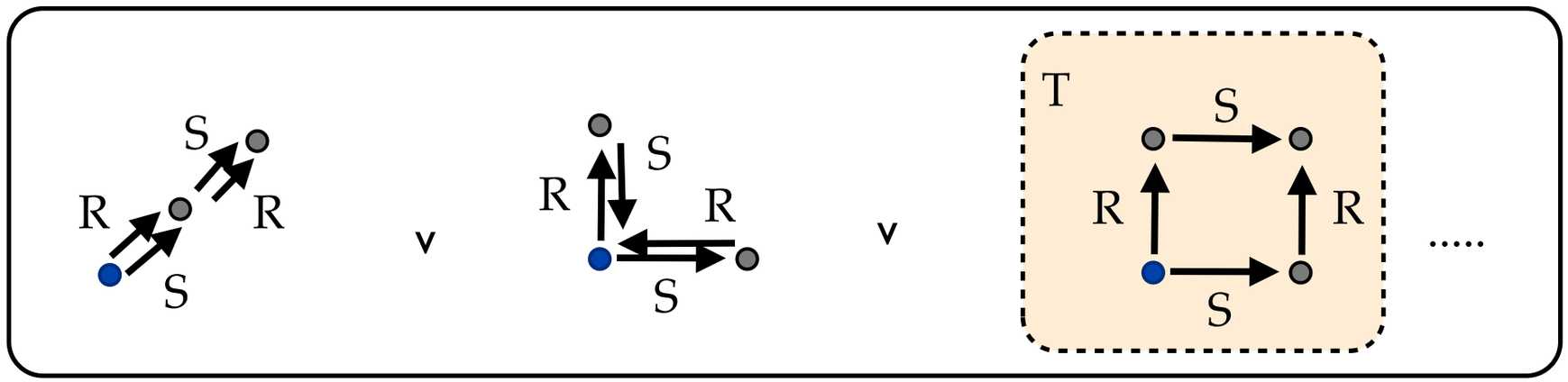

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  - The treeification Λ^{τ}_{q} can be expressed in GFO, and it "approximates" q:
    - By definition, Λ^{τ}_{q} implies q. Over acyclic structures, q and Λ^{τ}_{q} are equivalent.

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  - In the finite case, instead of M^* we use a Rosati-cover (a finite approximation of M^*).

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- Open question:
  - Craig interpolation for GNFO
  - Finite model-theoretic characterization of GNFO in terms of GN-bisimulations (characterization for $GNFO^k$ was shown by Martin Otto)