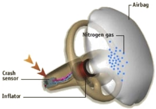
On the optimal reachability problem in weighted timed automata and games

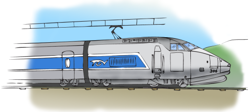
Patricia Bouyer-Decitre

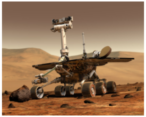
LSV, CNRS & ENS Cachan, France

Time-dependent systems

- We are interested in timed systems







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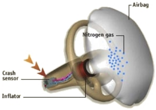
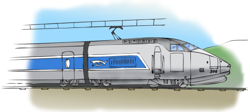
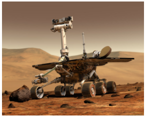


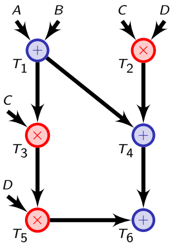




- ... and in their analysis and control

An example: The task graph scheduling problem

Compute $D\times(C\times(A+B))+(A+B)+(C\times D)$ using two processors:

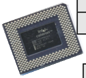


P_1 (fast):

time	
$+$	2 picoseconds
$\times$	3 picoseconds

energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



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[BFLM10] Bouyer, Fahrenberg, Larsen, Markey. Quantitative Analysis of Real-Time Systems using Priced Timed Automata *(Communication of the ACM)*.

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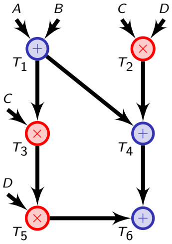

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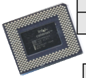

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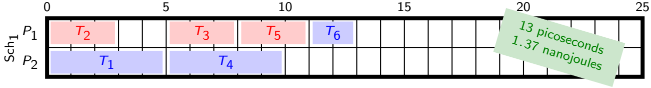

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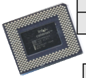

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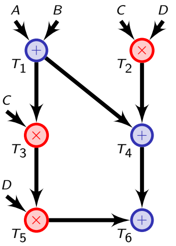

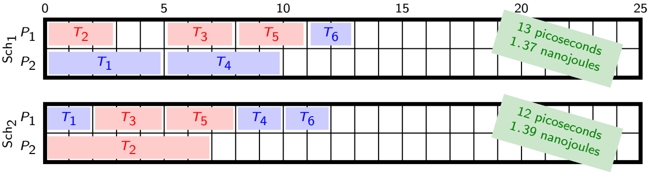

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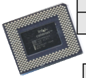

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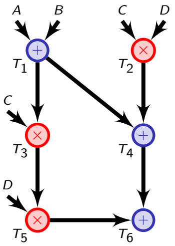

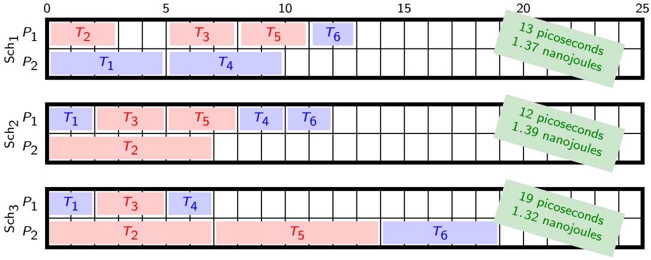

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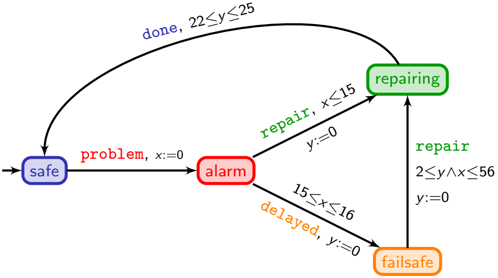
Outline

The model of timed automata



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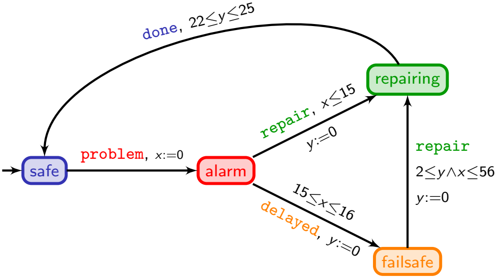


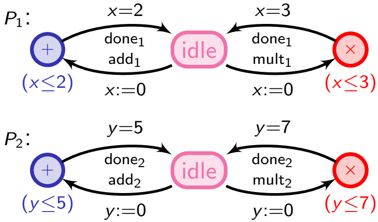
	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Modelling the task graph scheduling problem

Modelling the task graph scheduling problem

- Processors



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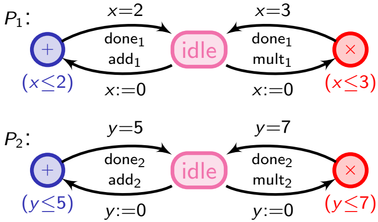

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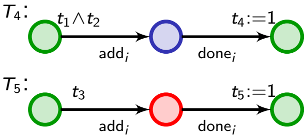

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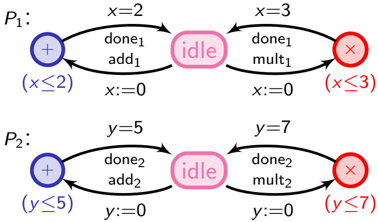

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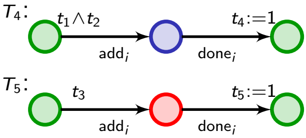

$\rightsquigarrow$ build the synchronized product of all these automata

$$(P_1 \parallel P_2) \parallel_s (T_1 \parallel T_2 \parallel \cdots \parallel T_6)$$

Modelling the task graph scheduling problem

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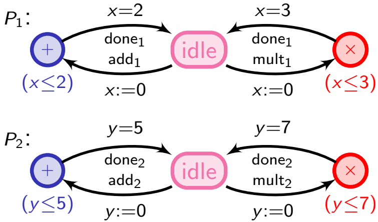

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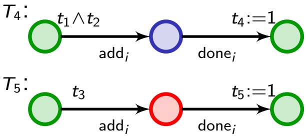

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A schedule: a path in the global system which reaches $t_1 \wedge \cdots \wedge t_6$

Modelling the task graph scheduling problem

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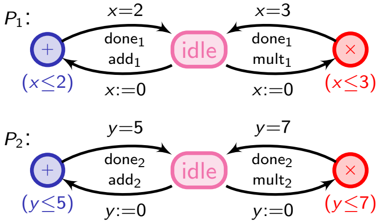

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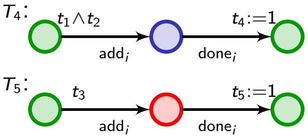

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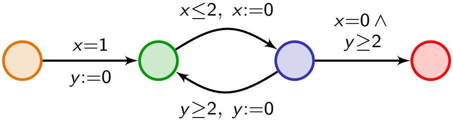
Questions one can ask

- Can the computation be made in no more than 10 time units?
- Is there a scheduling along which no processor is ever idle?
- ...

Analyzing timed automata



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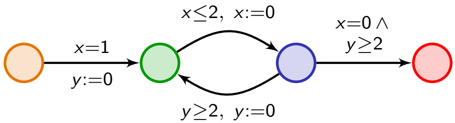

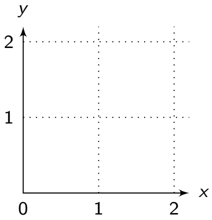

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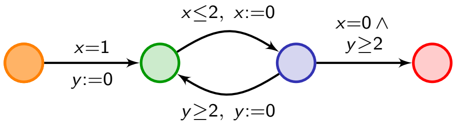

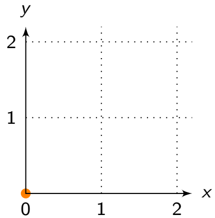

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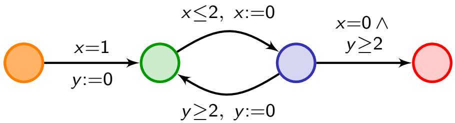

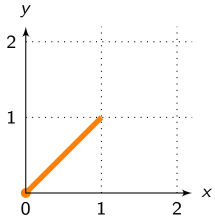

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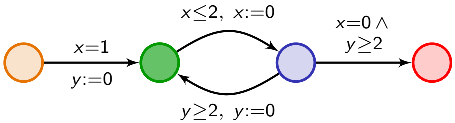

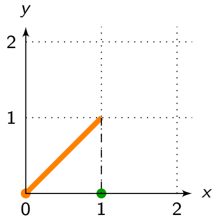

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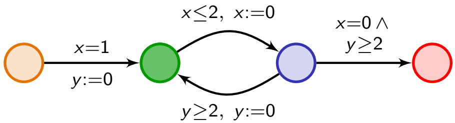

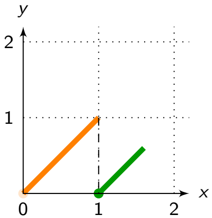

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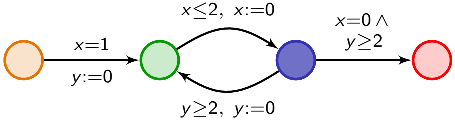

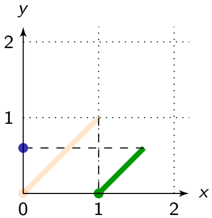

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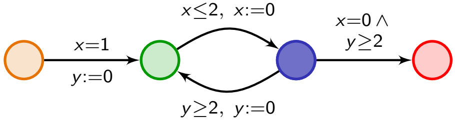

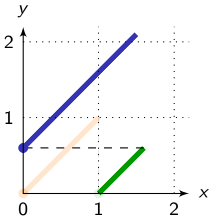

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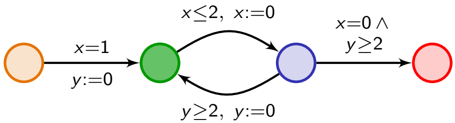

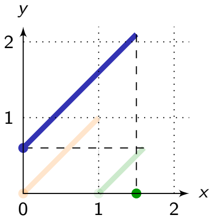

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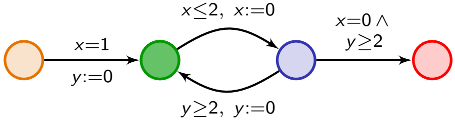

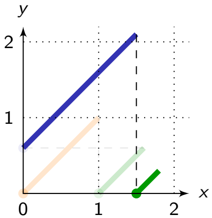

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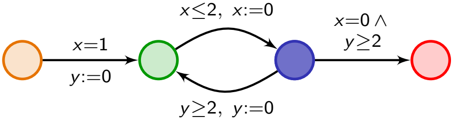

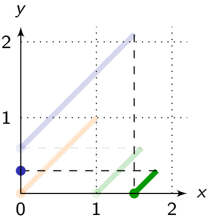

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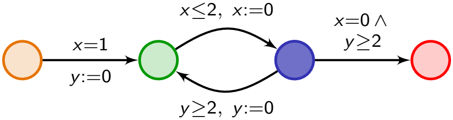

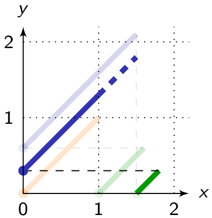

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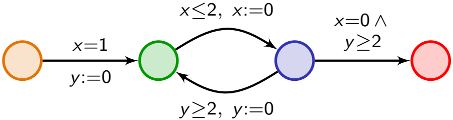

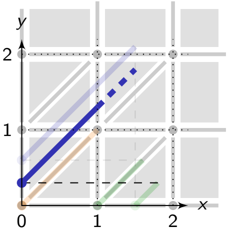

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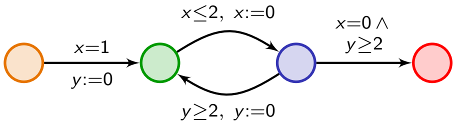
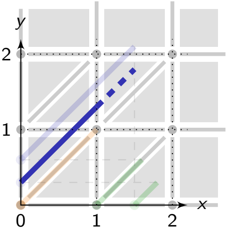




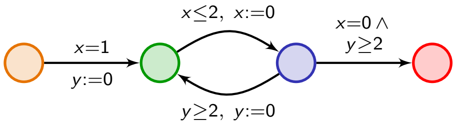
Theorem [AD94]

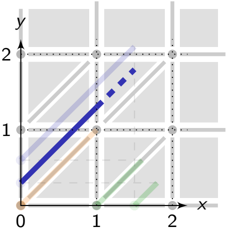
Reachability in timed automata is decidable (as well as many other important properties). It is PSPACE-complete.

- Technical tool: region abstraction

[AD94] Alur, Dill. A Theory of Timed Automata *(Theoretical Computer Science)*.

Analyzing timed automata





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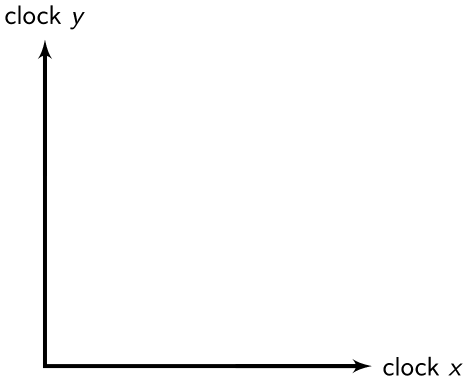
- Technical tool: region abstraction
- Efficient symbolic technics based on zones, implemented in tools

[AD94] Alur, Dill. A Theory of Timed Automata *(Theoretical Computer Science)*.

Technical tool: Region abstraction



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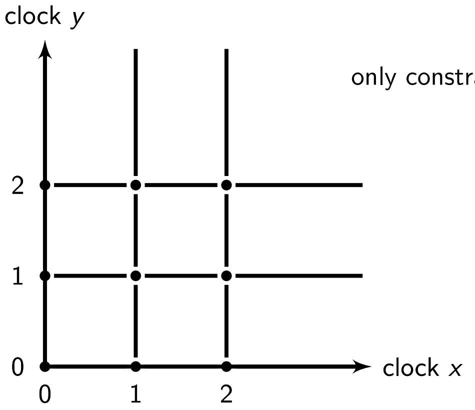
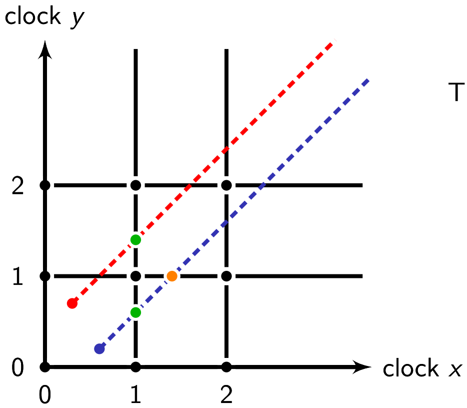


only constraints: $x \sim c$ with $c \in \{0, 1, 2\}$
$y \sim c$ with $c \in \{0, 1, 2\}$

- "compatibility" between regions and constraints

Technical tool: Region abstraction



The path 

- can be fired from ●
- cannot be fired from ●

- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

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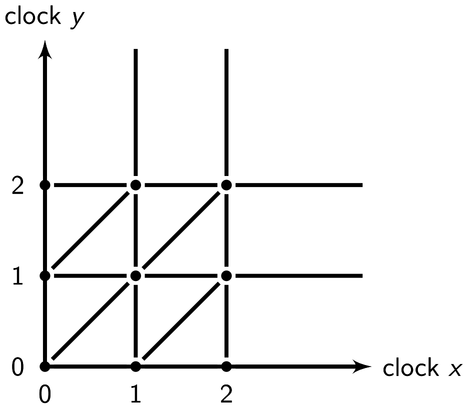

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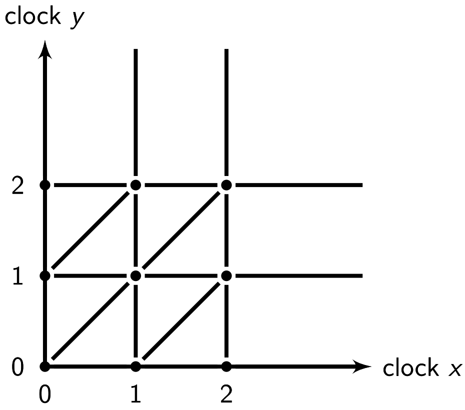
- “compatibility” between regions and time elapsing

Technical tool: Region abstraction



- "compatibility" between regions and constraints
- "compatibility" between regions and time elapsing

$\rightsquigarrow$ This is a finite time-abstract bisimulation!

Time-abstract bisimulation

This is a relation between • and • such that:

Time-abstract bisimulation

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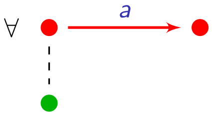

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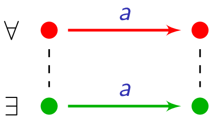

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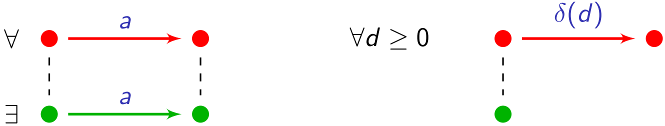

Time-abstract bisimulation

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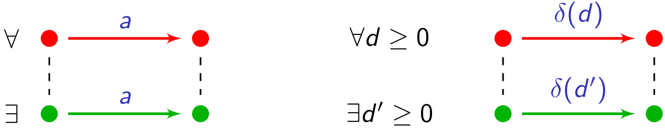

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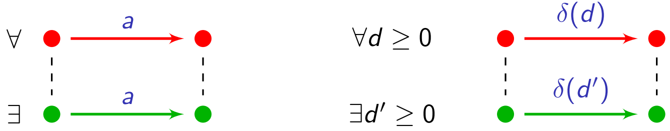

... and vice-versa (swap • and •).

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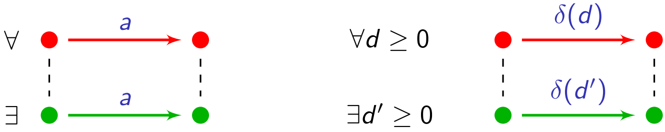

... and vice-versa (swap • and •).

Consequence

$$\forall \quad (\ell_1, v_1) \xrightarrow{d_1, a_1} (\ell_2, v_2) \xrightarrow{d_2, a_2} (\ell_3, v_3) \xrightarrow{d_3, a_3} \quad \cdots$$

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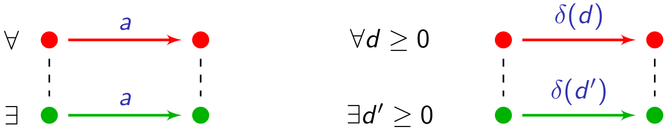

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$$(\ell_1, R_1) \xrightarrow{a_1} (\ell_2, R_2) \xrightarrow{a_2} (\ell_3, R_3) \xrightarrow{a_3} \cdots \quad \text{with } v_i \in R_i$$

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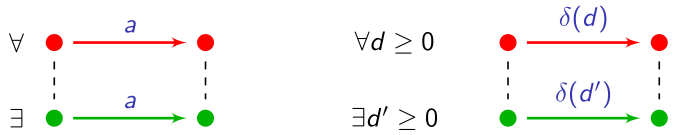

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$$\downarrow \qquad \downarrow \qquad \downarrow$$

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$\forall v_1' \in R_1$

Time-abstract bisimulation

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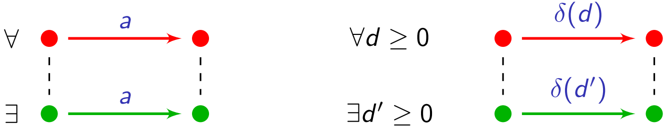

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$$
\begin{array}{cccccccccc}
\forall & (\ell_1, v_1) & \xrightarrow{d_1, a_1} & (\ell_2, v_2) & \xrightarrow{d_2, a_2} & (\ell_3, v_3) & \xrightarrow{d_3, a_3} & \cdots & \\
 & \downarrow & & \downarrow & & \downarrow & & & \\
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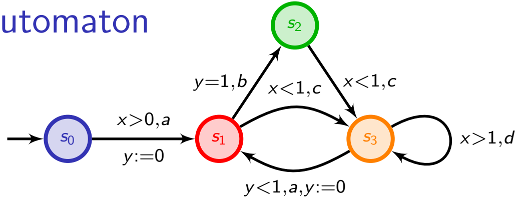
 & \downarrow & & \downarrow & & \downarrow & & & \\
\forall v'_1 \in R_1 \; \exists & (\ell_1, v'_1) & \xrightarrow{d'_1, a_1} & (\ell_2, v'_2) & \xrightarrow{d'_2, a_2} & (\ell_3, v'_3) & \xrightarrow{d'_3, a_3} & \cdots & \text{with } v'_i \in R_i
\end{array}
$$

The region automaton



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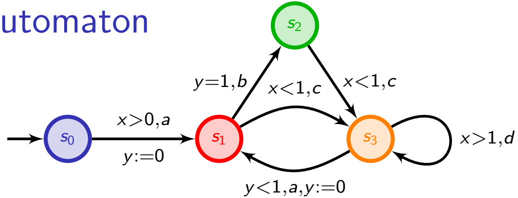

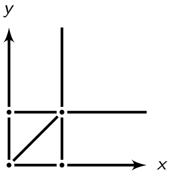

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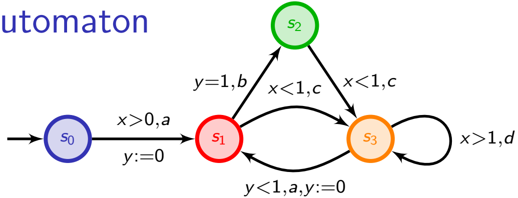

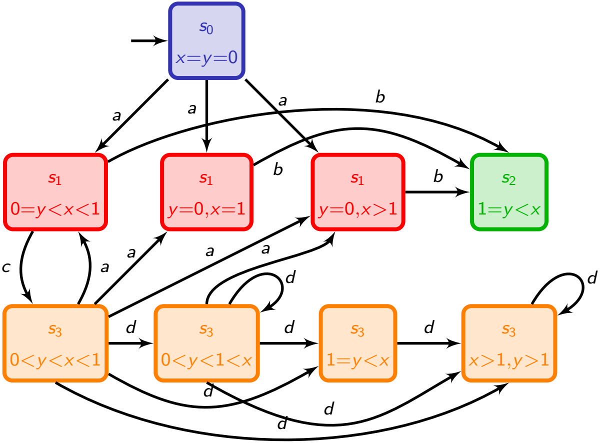

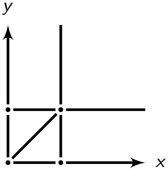

Outline

Modelling resources in timed systems

- System resources might be relevant and even crucial information

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  - energy consumption,
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$\leadsto$ timed automata are not powerful enough!

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- A possible solution: use hybrid automata

  a discrete control (the mode of the system)
  \+ continuous evolution of the variables within a mode

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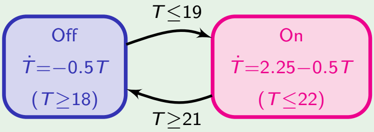
The thermostat example



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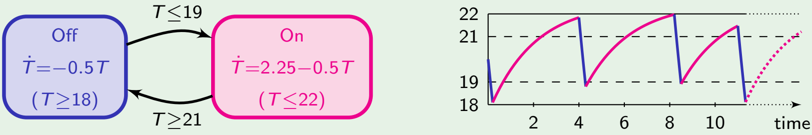
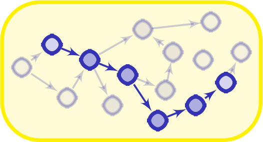


Ok...



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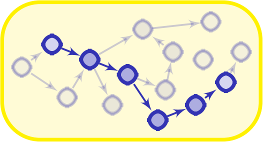

Easy...

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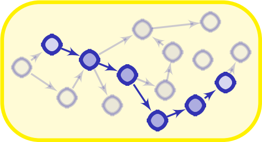

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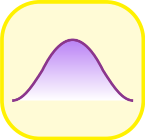

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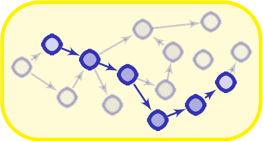

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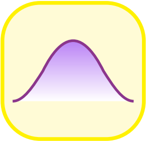
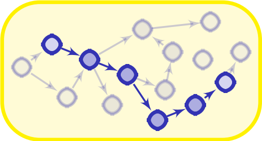


Easy...

Ok... but?



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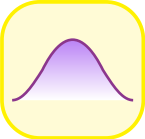

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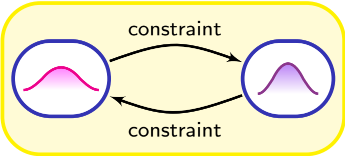

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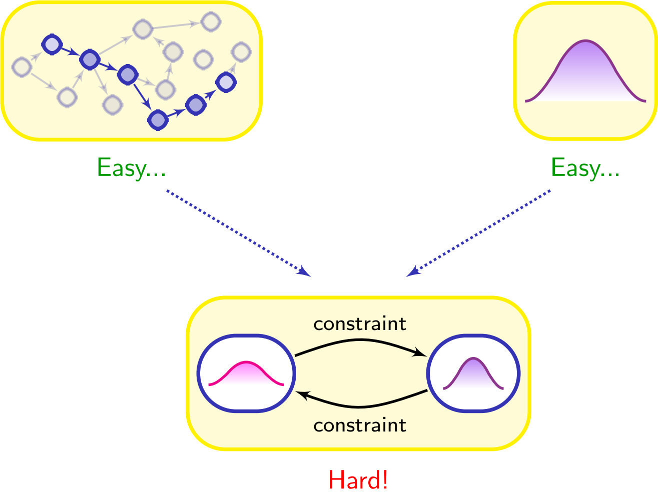

Modelling resources in timed systems

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Theorem [HKPV95]

The reachability problem is undecidable in hybrid automata. Even for the simplest, the so-called stopwatch automata (clocks can be stopped).

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.

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- An alternative: weighted/priced timed automata [ALP01,BFH+01]
  $\rightsquigarrow$ hybrid variables do not constrain the system
  hybrid variables are observer variables

[HKPV95] Henzinger, Kopke, Puri, Varaiya. What's decidable wbout hybrid automata? *(SToC'95)*.
[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
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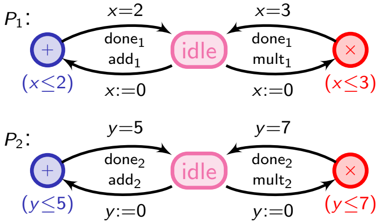
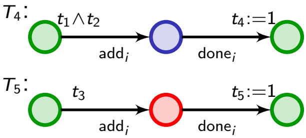
Modelling the task graph scheduling problem

- Processors



- Tasks



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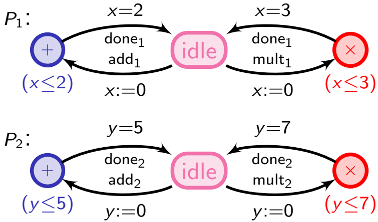

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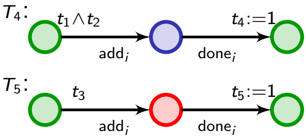
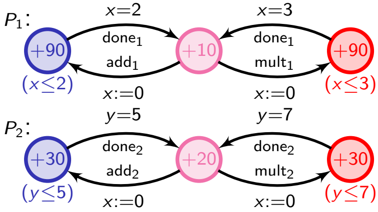
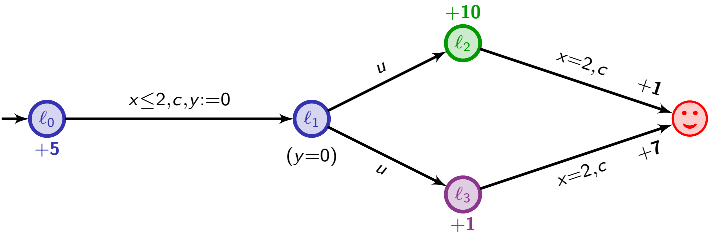


- Modelling energy



A good schedule is a path in the product automaton with a low cost

Weighted/priced timed automata [ALP01,BFH+01]



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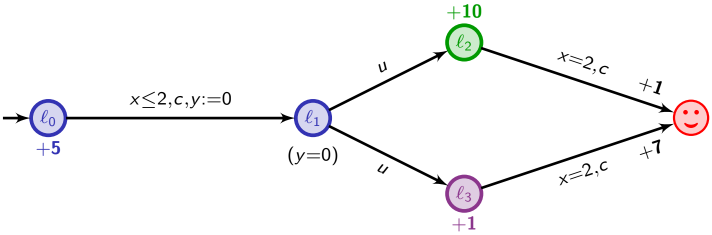

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

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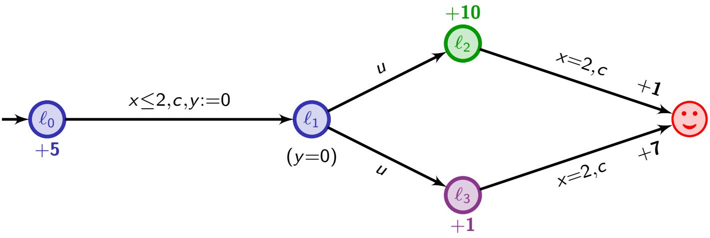

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost :

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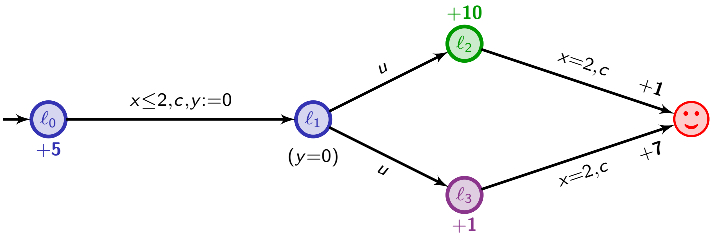

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5

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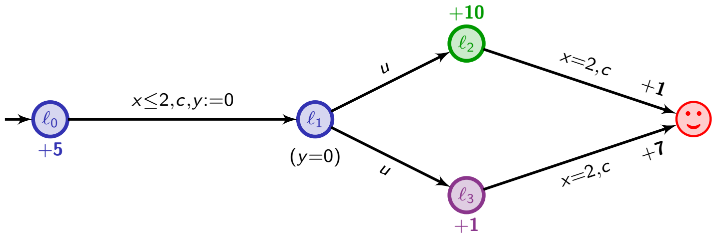

	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
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y	0		1.3		0		0		0.7		

cost : 6.5 + 0

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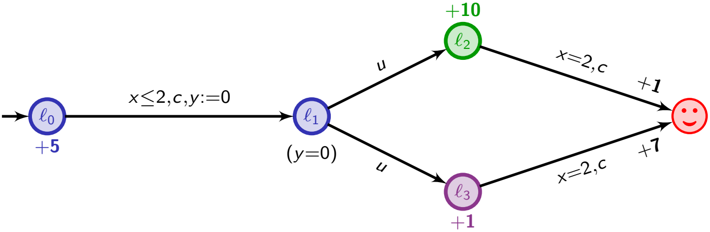

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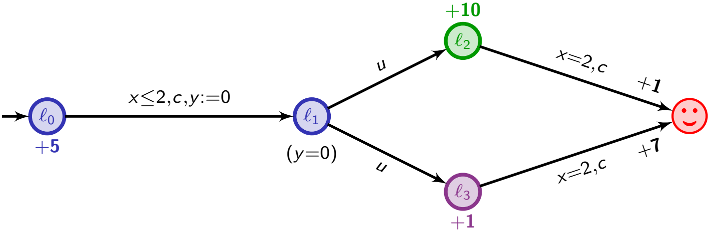

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x	0		1.3		1.3		1.3		2		
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cost :		6.5	+	0	+	0	+	0.7			

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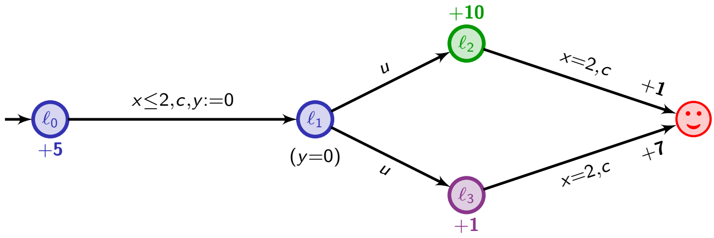

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x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		
cost :		6.5	+	0	+	0	+	0.7	+	7	

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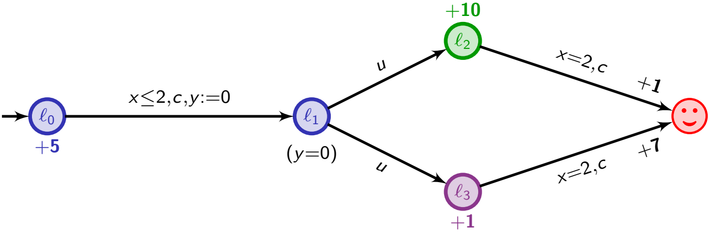
Weighted/priced timed automata [ALP01,BFH+01]



	ℓ_0	$\xrightarrow{1.3}$	ℓ_0	$\xrightarrow{c}$	ℓ_1	$\xrightarrow{u}$	ℓ_3	$\xrightarrow{0.7}$	ℓ_3	$\xrightarrow{c}$	☺
x	0		1.3		1.3		1.3		2		
y	0		1.3		0		0		0.7		

cost : 6.5 + 0 + 0 + 0.7 + 7 = 14.2

[ALP01] Alur, La Torre, Pappas. Optimal paths in weighted timed automata *(HSCC'01)*.
[BFH+01] Behrmann, Fehnker, Hune, Larsen, Pettersson, Romijn, Vaandrager. Minimum-cost reachability in priced timed automata *(HSCC'01)*.

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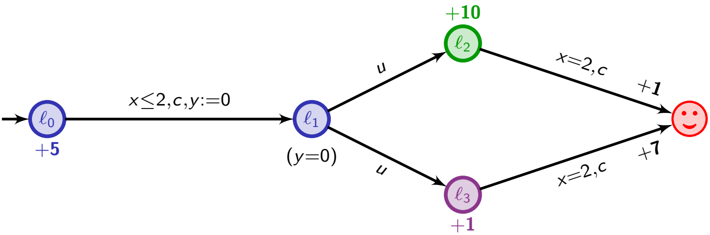

Question: what is the optimal cost for reaching 🙂 ?

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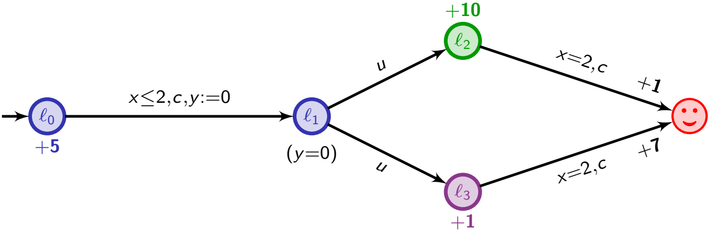

Question: what is the optimal cost for reaching ☺?

$$5t + 10(2 - t) + 1$$

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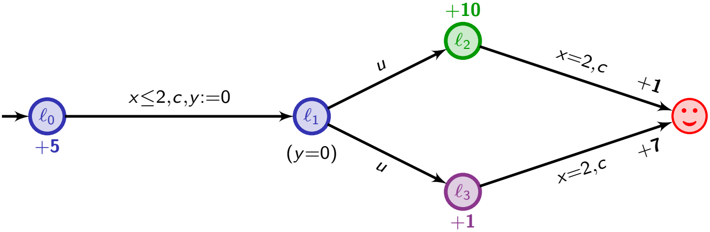

Question: what is the optimal cost for reaching ☺ ?

$$5t + 10(2 - t) + 1 \ , \ 5t + (2 - t) + 7$$

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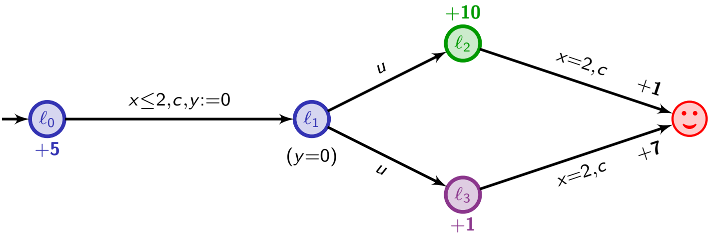

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$$\min\left(5t + 10(2-t) + 1 \ ,\ 5t + (2-t) + 7 \right)$$

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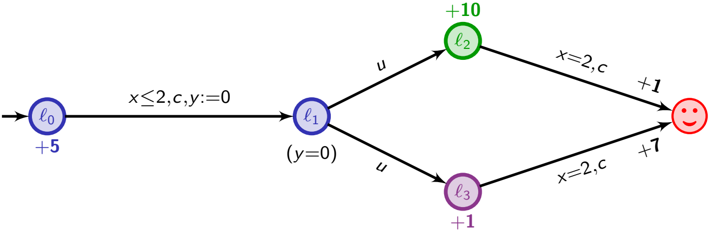

Question: what is the optimal cost for reaching ☺?

$$\inf_{0 \leq t \leq 2} \min \left(5t + 10(2-t) + 1 \; , \; 5t + (2-t) + 7 \right) = 9$$

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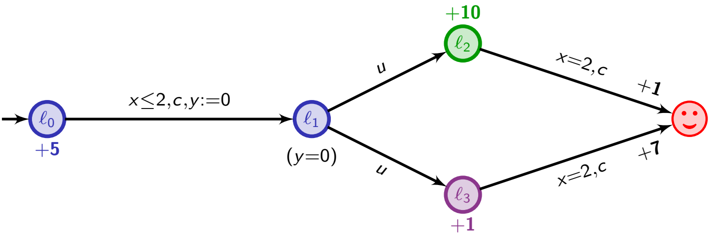

Question: what is the optimal cost for reaching ☺ ?

$$\inf_{0\leq t\leq 2} \min\left(\, 5t+10(2-t)+1 \,,\, 5t+(2-t)+7 \,\right) = 9$$

$\rightsquigarrow$ *strategy:* leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

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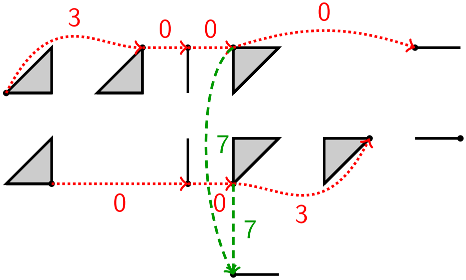
Optimal-cost reachability

Theorem [ALP01,BFH+01,BBBR07]

In weighted timed automata, the optimal cost is an integer and can be computed in PSPACE.

- Technical tool: a refinement of the regions, the corner-point abstraction



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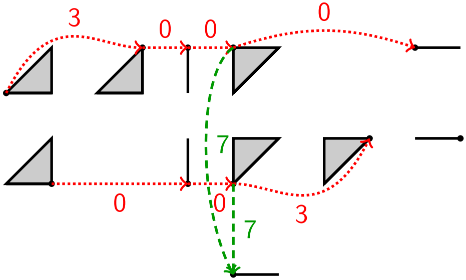

- Symbolic technics based on priced zones

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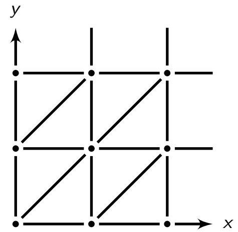
Technical tool: the corner-point abstraction



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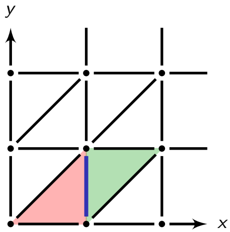

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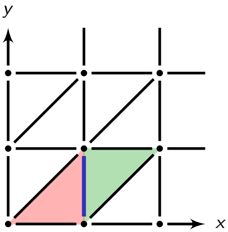
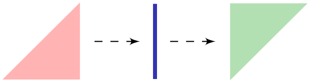


Abstract time successors:



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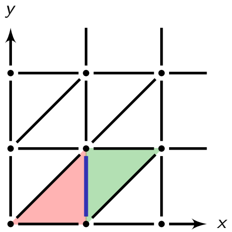

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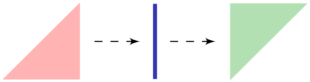
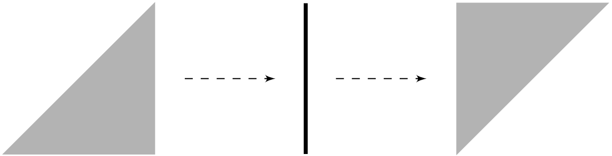


Concrete time successors:



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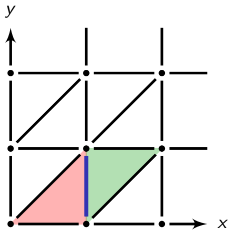

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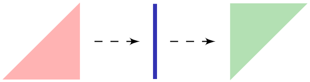

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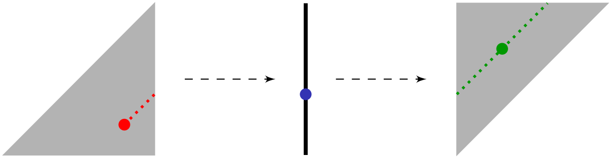

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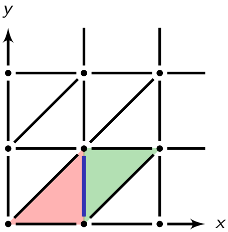

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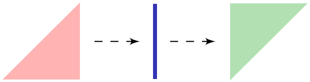

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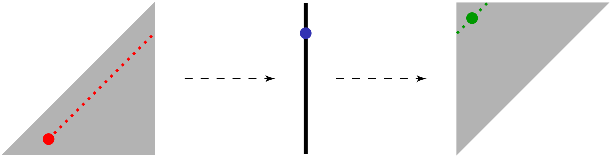

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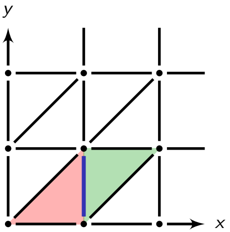

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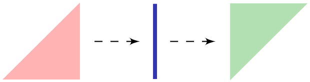

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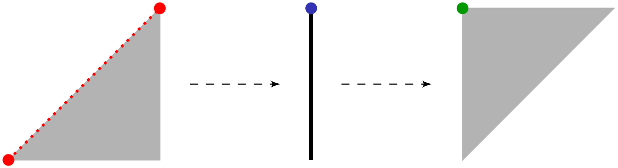

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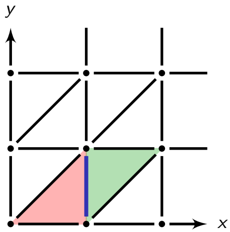

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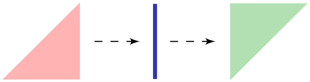

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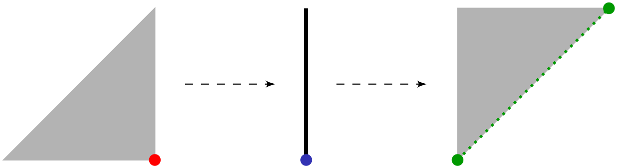

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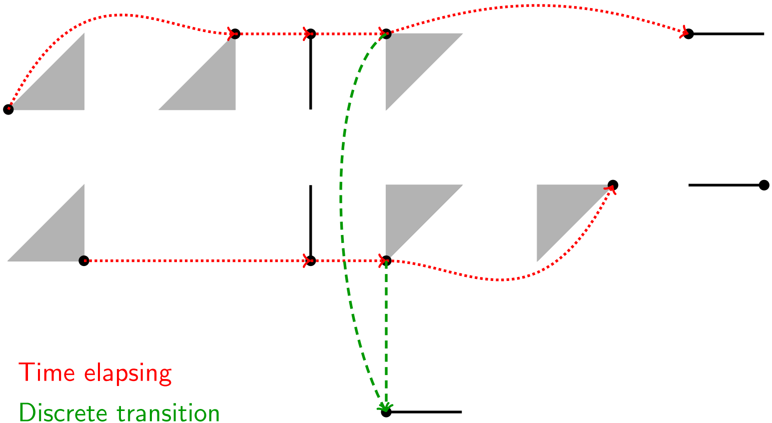
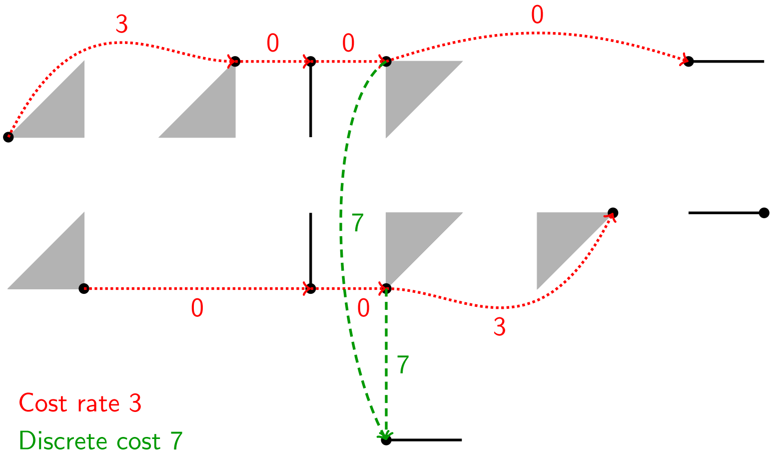


Time elapsing

Discrete transition

Technical tool: the corner-point abstraction



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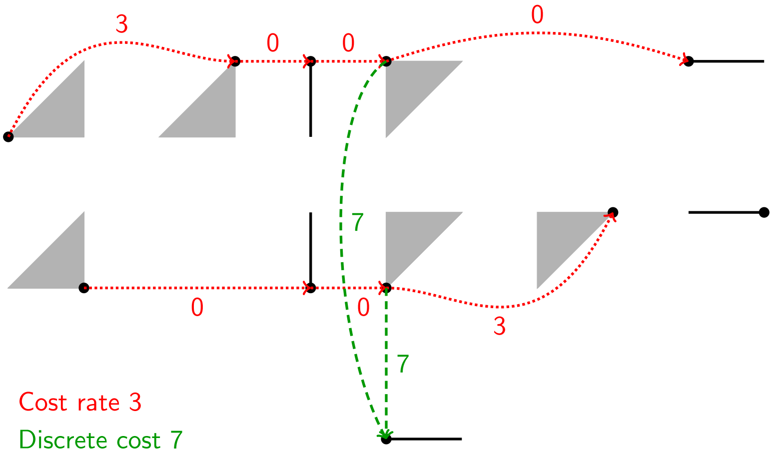
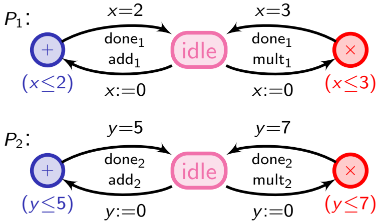


Optimal cost in the weighted graph
= optimal cost in the weighted timed automaton!

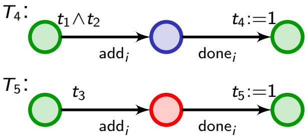
Outline

Modelling the task graph scheduling problem

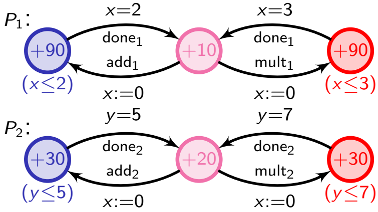
- Processors



- Tasks



- Modelling energy



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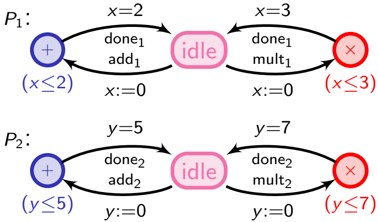
- Processors



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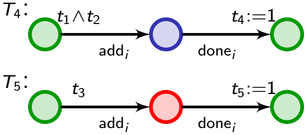

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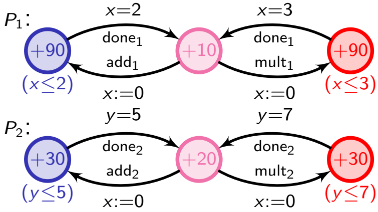
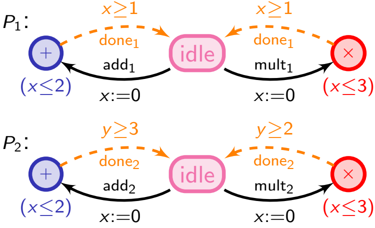


- Modelling uncertainty



Modelling the task graph scheduling problem

- Processors

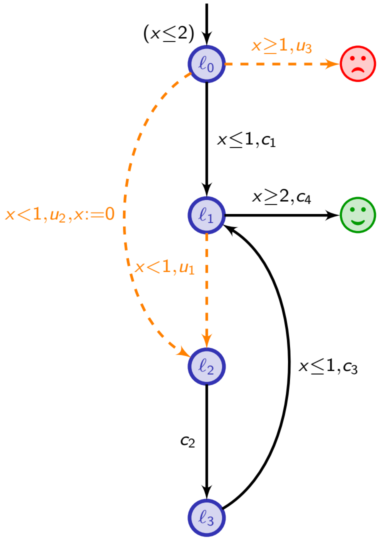
- Tasks

A (good) schedule is a strategy in the product game (with a low cost)

- Modelling energy

- Modelling uncertainty

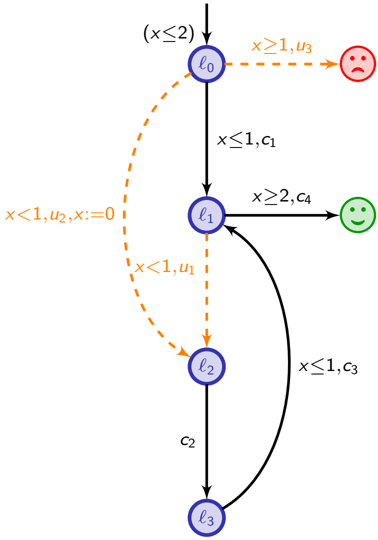
An example of a timed game



Rule of the game

- Aim: avoid ☹ and reach ☺

An example of a timed game



Rule of the game

- Aim: avoid ☹ and reach ☺
- How do we play? According to a strategy:

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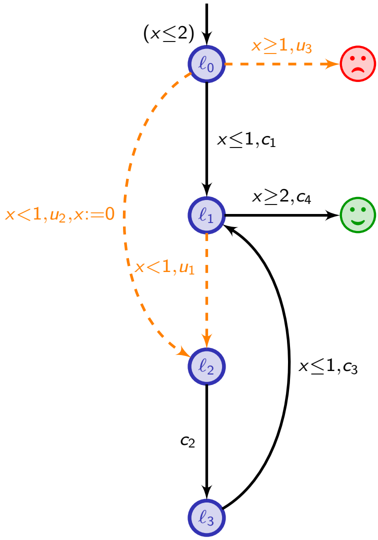

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$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

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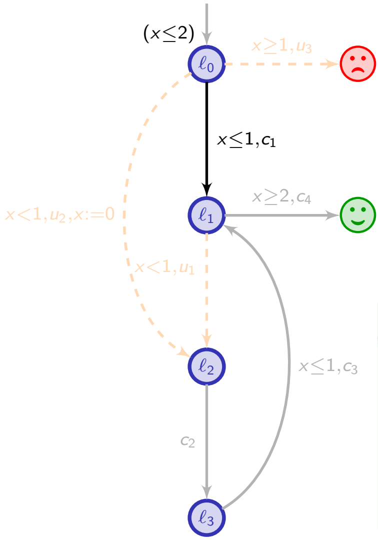

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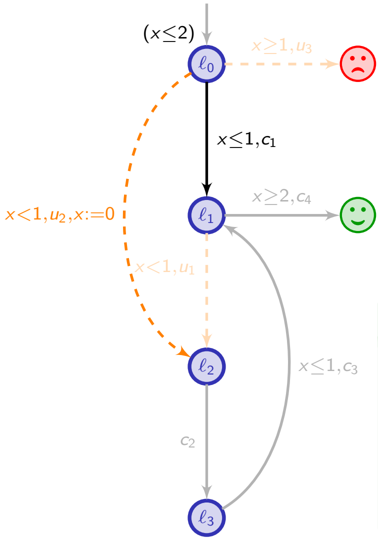
A (memoryless) winning strategy

- from $(\ell_0, 0)$, play $(0.5, c_1)$

An example of a timed game



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  $\rightsquigarrow$ can be preempted by u_2

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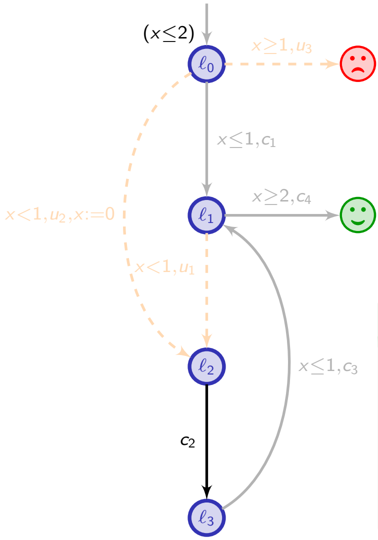

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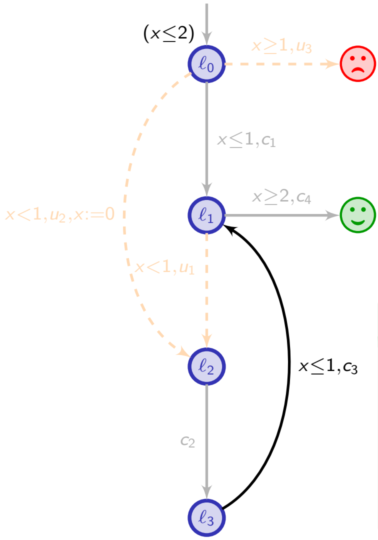
- from $(\ell_0, 0)$, play $(0.5, c_1)$
  $\rightsquigarrow$ can be preempted by u_2
- from $(\ell_2, \star)$, play $(1 - \star, c_2)$

An example of a timed game



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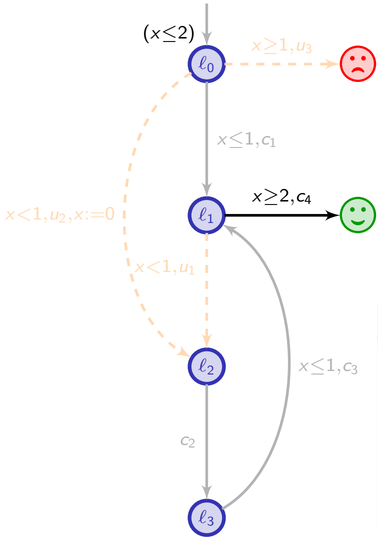

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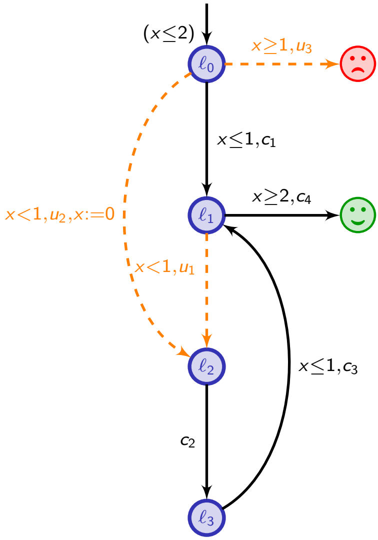
- from $(\ell_3, 1)$, play $(0, c_3)$
- from $(\ell_1, 1)$, play $(1, c_4)$

An example of a timed game



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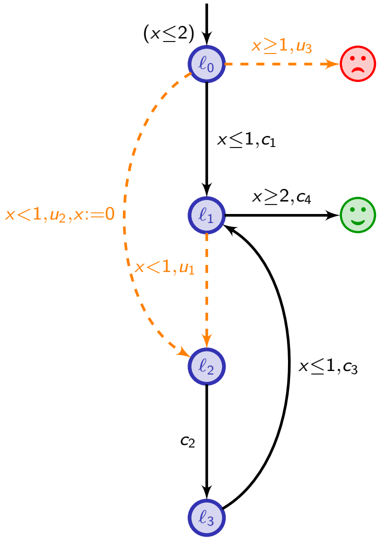
Problems to be considered

An example of a timed game



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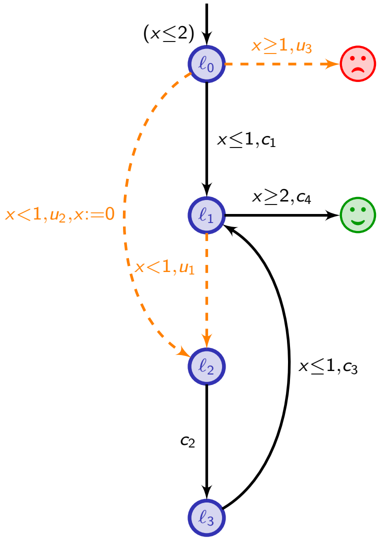
Problems to be considered

- Does there exist a winning strategy?

An example of a timed game



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- Aim: avoid ☹ and reach ☺
- How do we play? According to a strategy:

$$f : \text{history} \mapsto (\text{delay}, \text{cont. transition})$$

Problems to be considered

- Does there exist a winning strategy?
- If yes, compute one (as simple as possible).

Decidability of timed games

Theorem [AMPS98,HK99]

Reachability and safety timed games are decidable and EXPTIME-complete. Furthermore memoryless and "region-based" strategies are sufficient.

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
[HK99] Henzinger, Kopke. Discrete-time control for rectangular hybrid automata *(Theoretical Computer Science)*.

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$\rightsquigarrow$ classical regions are sufficient for solving such problems
a region-closed attractor can be computed

[AMPS98] Asarin, Maler, Pnueli, Sifakis. Controller synthesis for timed automata *(SSC'98)*.
[HK99] Henzinger, Kopke. Discrete-time control for rectangular hybrid automata *(Theoretical Computer Science)*.

Decidability of timed games

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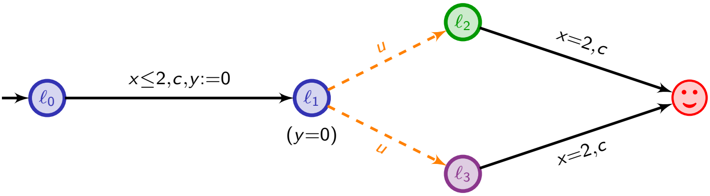
Theorem [AM99,BHPR07,JT07]

Optimal-time reachability timed games are decidable and EXPTIME-complete.

[AM99] Asarin, Maler. As soon as possible: time optimal control for timed automata *(HSCC'99)*.
[BHPR07] Brihaye, Henzinger, Prabhu, Raskin. Minimum-time reachability in timed games *(ICALP'07)*.
[JT07] Jurdziński, Trivedi. Reachability-time games on timed automata *(ICALP'07)*.

Outline

A simple timed game



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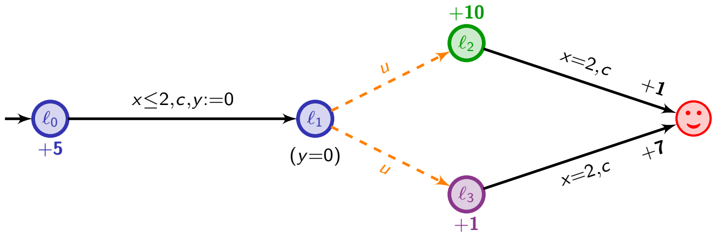

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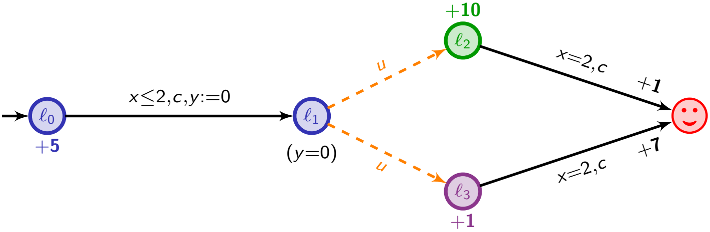

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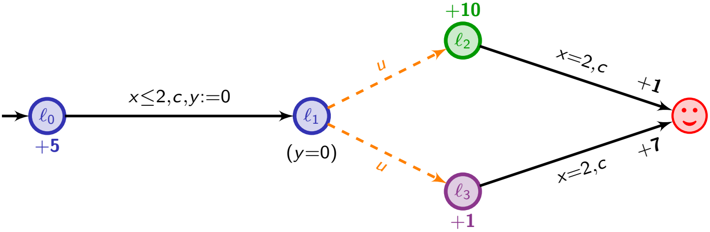

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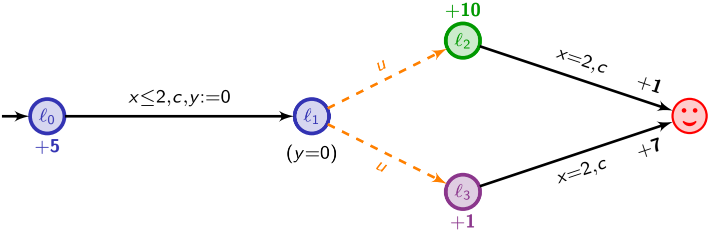
$$5t + 10(2 - t) + 1$$

A simple weighted timed game



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$$5t + 10(2-t) + 1 \ , \ 5t + (2-t) + 7$$

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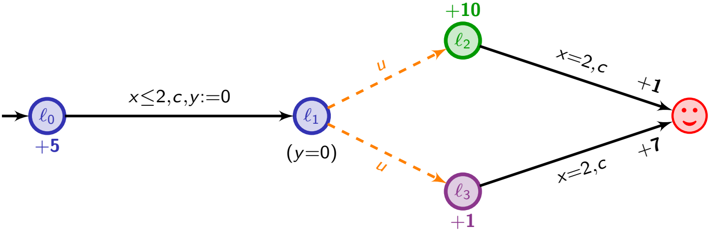

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$$\max\left(\ 5t + 10(2-t) + 1\ ,\ 5t + (2-t) + 7\ \right)$$

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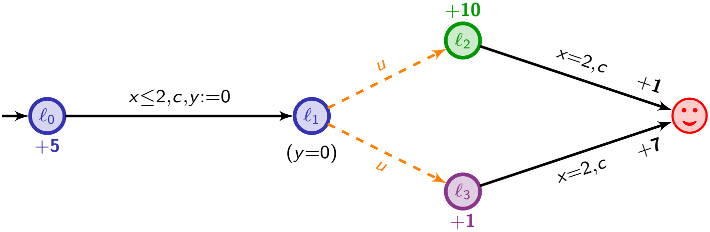

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$$\inf_{0 \leq t \leq 2} \max \left(5t + 10(2-t) + 1 \,,\, 5t + (2-t) + 7 \right) = 14 + \frac{1}{3}$$

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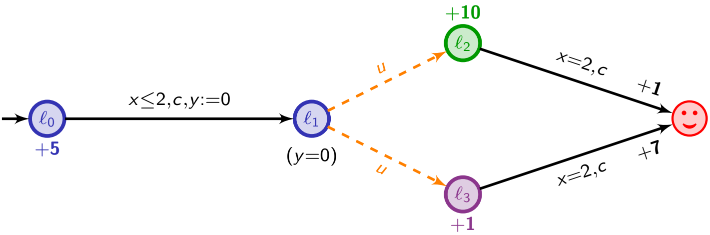

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$\rightsquigarrow$ *strategy:* wait in ℓ_0, and when $t = \frac{4}{3}$, go to ℓ_1

Optimal reachability in weighted timed games (1)

This topic has been fairly hot these last fifteen years...

[LMM02,ABM04,BCFL04,BBR05,BBM06,BLMR06,Rut11,HIM13,BGK+14]

[LMM02] La Torre, Mukhopadhyay, Murano. Optimal-reachability and control for acyclic weighted timed automata *(TCS@02)*.
[ABM04] Alur, Bernardsky, Madhusudan. Optimal reachability in weighted timed games *(ICALP'04)*.
[BCFL04] Bouyer, Cassez, Fleury, Larsen. Optimal strategies in priced timed game automata *(FSTTCS'04)*.
[BBR05] Brihaye, Bruyère, Raskin. On optimal timed strategies *(FORMATS'05)*.
[BBM06] Bouyer, Brihaye, Markey. Improved undecidability results on weighted timed automata *(Information Processing Letters)*.
[BLMR06] Bouyer, Larsen, Markey, Rasmussen. Almost-optimal strategies in one-clock priced timed automata *(FSTTCS'06)*.
[Rut11] Rutkowski. Two-player reachability-price games on single-clock timed automata *(QAPL'11)*.
[HIM13] Hansen, Ibsen-Jensen, Miltersen. A faster algorithm for solving one-clock priced timed games *(CONCUR'13)*.
[BGK+14] Brihaye, Geeraerts, Krishna, Manasa, Monmege, Trivedi. Adding Negative Prices to Priced Timed Games *(CONCUR'14)*.

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Tree-like weighted timed games can be solved in 2EXPTIME.

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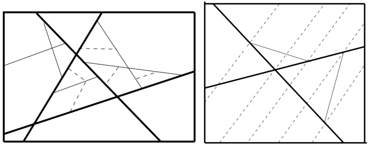
[LMM02]

Tree-like weighted timed games can be solved in 2EXPTIME.

[ABM04,BCFL04]

Depth-k weighted timed games can be solved in EXPTIME. There is a symbolic algorithm to solve weighted timed games **with a strongly non-Zeno cost**.



Optimal reachability in weighted timed games (2)

[BBR05,BBM06,BJM15]

In weighted timed games, the optimal cost (and the value) cannot be computed, as soon as games have three clocks or more.

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[BLMR06,Rut11,HIM13,BGK+14]

Turn-based optimal timed games are decidable in EXPTIME (resp. PTIME) when automata have a single clock (resp. with two rates). They are PTIME-hard.

Computing the optimal cost: why is that hard?

Given two clocks x and y, we can check whether $y = 2x$.

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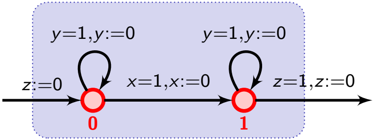
$\text{Add}^+(x)$



The cost is increased by x_0

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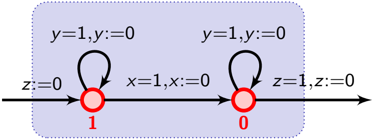

The cost is increased by $1-x_0$

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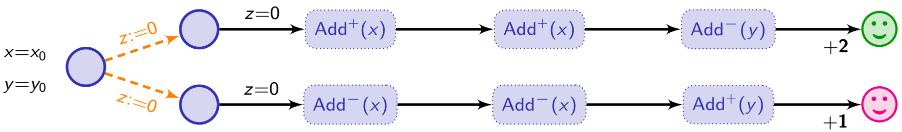

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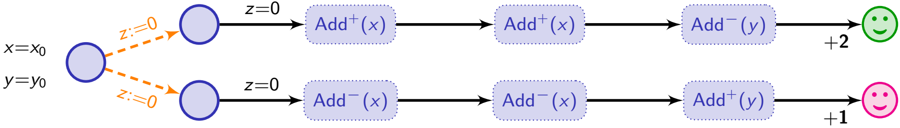

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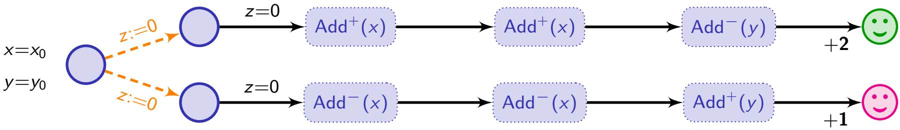

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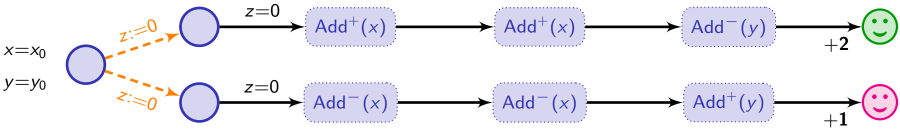

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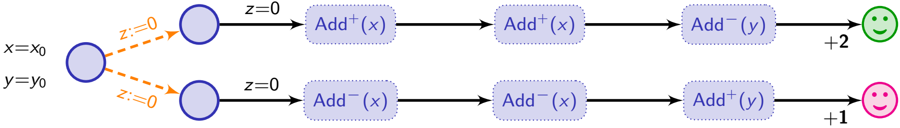

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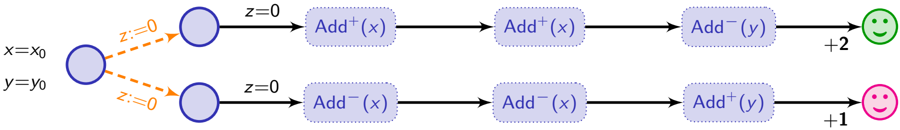

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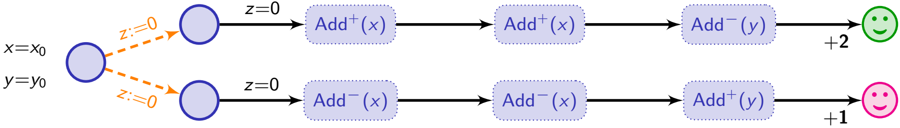

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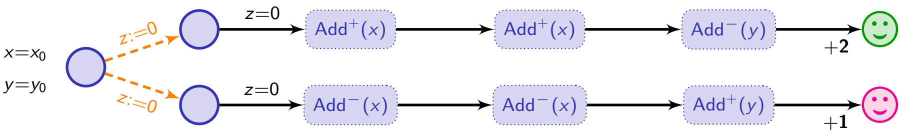

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- Player 1 has a winning strategy with cost ≤ 3 iff $y_0 = 2x_0$

Computing the optimal cost: why is that hard?

Player 1 will simulate a two-counter machine:

- each instruction is encoded as a module;
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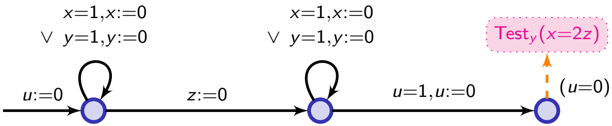
Globally, $(x \leq 1, y \leq 1, u \leq 1)$



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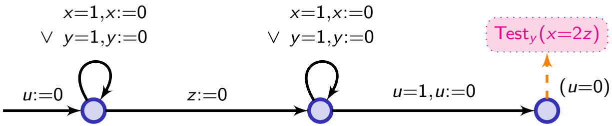

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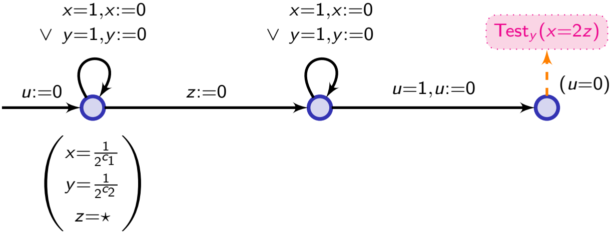

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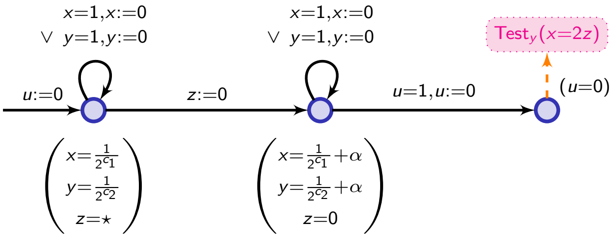

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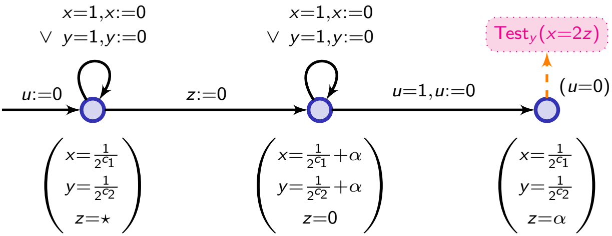

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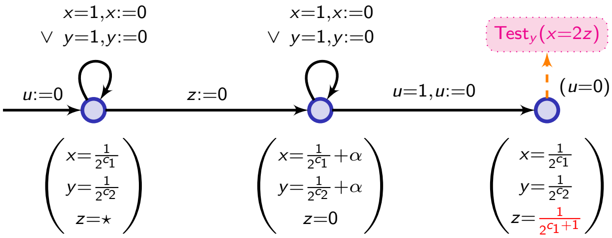

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... when cost is strongly non-zeno. [AM04,BCFL04]

There is $\kappa > 0$ s.t. for every region cycle C, for every real run ϱ read on C,

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- Almost-optimality in practice should be sufficient
- Even when we know how to compute the value, we are only able to synthesize almost-optimal strategies...

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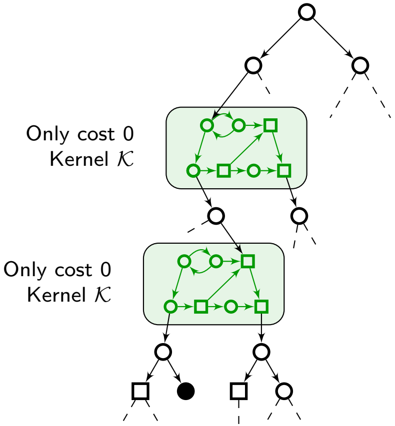
Idea of the proof: Semi-unfolding



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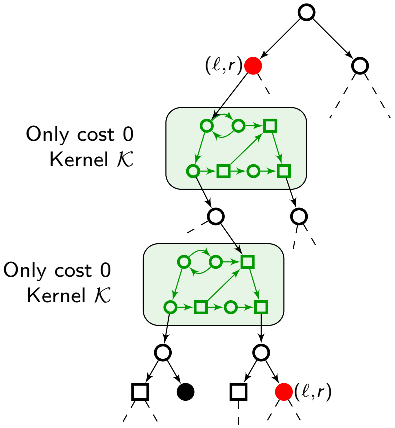

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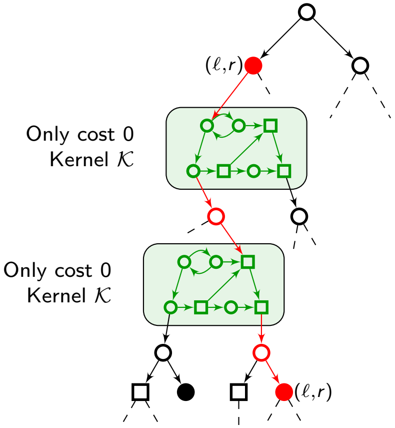

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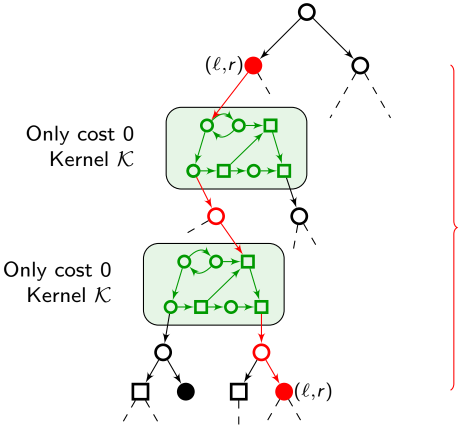

Hypothesis:
cost > 0 implies cost $\geq \kappa$

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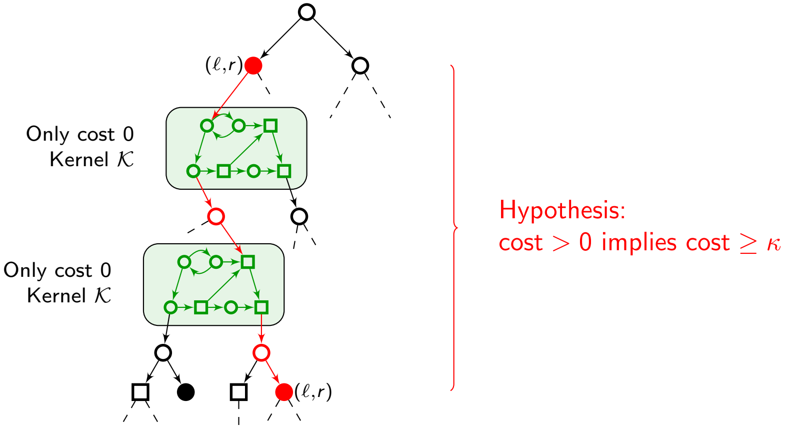
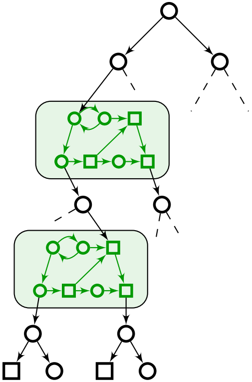


Conclusion: we can stop unfolding the game after finitely many steps

Approximation scheme



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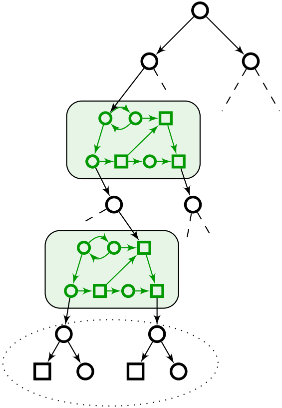

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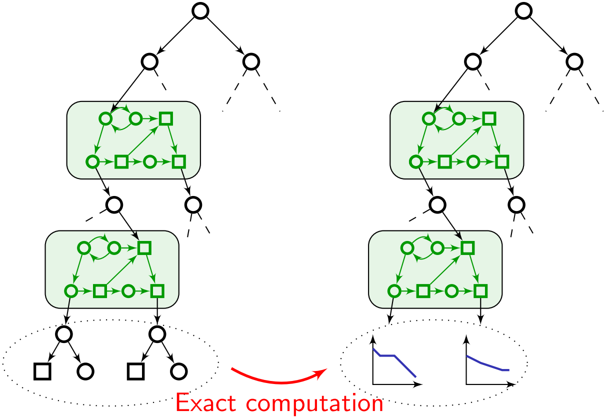

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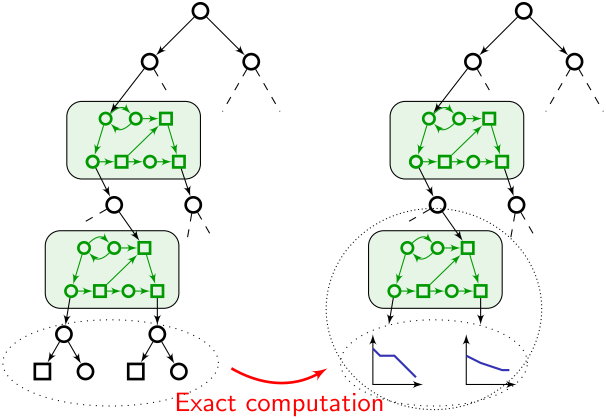

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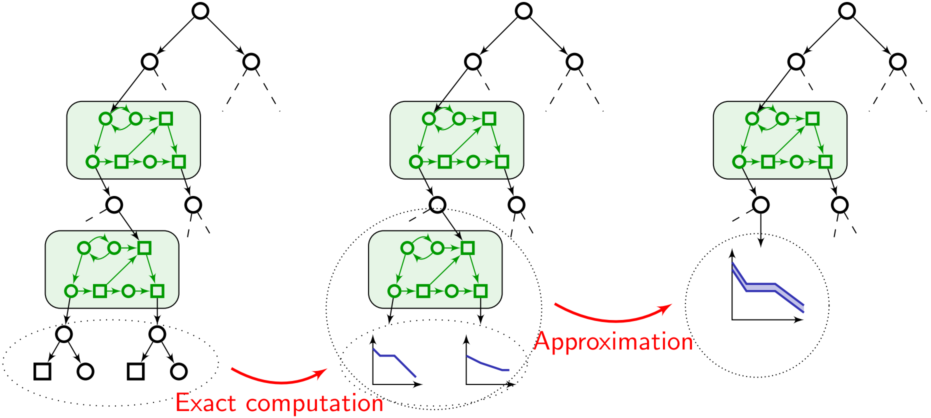

Outline

Tools for (weighted) timed automata and games

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- Tool-suite Uppaal, developed in Aalborg (Denmark) and originally Uppsala (Sweden) since 1995
  - Uppaal for timed automata
  - Uppaal-TiGa for timed games
  - Uppaal-Cora for weighted timed automata

Uppaal url: `http://www.uppaal.org`

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- Tool-suite Uppaal, developed in Aalborg (Denmark) and originally Uppsala (Sweden) since 1995
- Our new tool TiAMo, developed by Maximilien Colange (LSV), using code by Ocan Sankur (IRISA)

TiAMo = Timed Automata Model-checker

Uppaal url: http://www.uppaal.org
TiAMo url: https://git.lsv.fr/colange/tiamo

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Model-checker

- Timed automata:
  (time-optimal) reachability
- Weighted timed automata:
  optimal rechability

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TiAMo = Timed Automata Model-checker

- Timed automata: (time-optimal) reachability
- Weighted timed automata: optimal rechability
- Aims at being a platform for experiments (open source!)
- Aims at asserting and comparing algorithms

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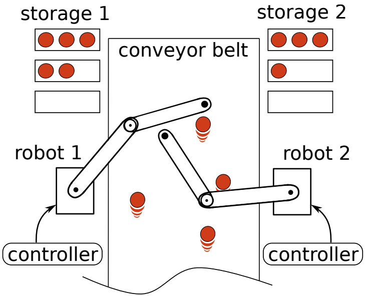
Outline

1. Timed automata
2. Weighted timed automata
3. Timed games
4. Weighted timed games
5. Tools
6. Towards applying all this theory to robotic systems
7. Conclusion

Example problem, objective and approach



[BMPS15] Bouyer, Markey, Perrin, Schlehuber-Caissier. Timed-Automata Abstraction of Switched Dynamical Systems Using Control Funnels *(FORMATS'15)*.

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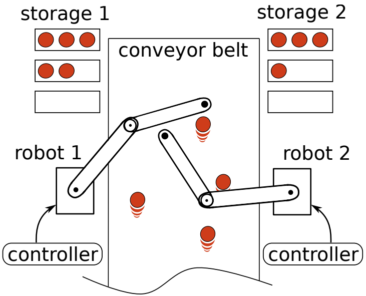

- Infinitely many configurations
- Complex behaviour
- Mechanical constraints

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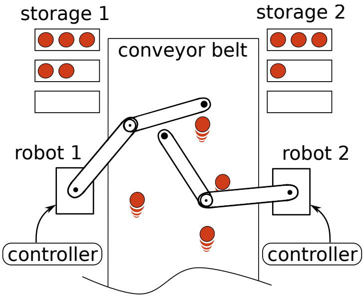

Goal: Synthesize a controller:

- Which robot handles an object
- How to avoid collision
- Don't miss any object

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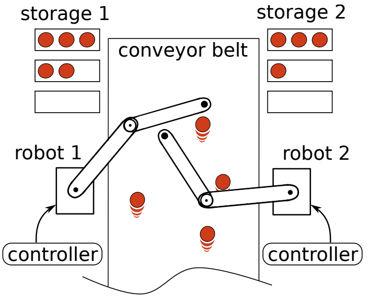

- Infinitely many configurations
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Goal: Synthesize a controller:

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Approach:

- Discretization of the behaviour via a fixed set of continuous controllers
- Create an abstraction and use previous results

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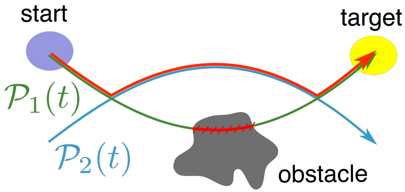
Our approach

Simplistic idea: fixed set of reference trajectories + property



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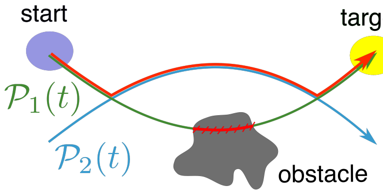
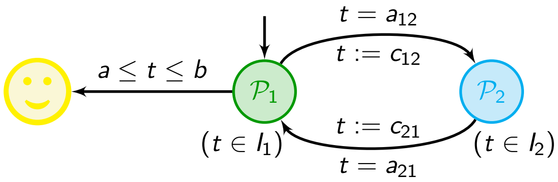
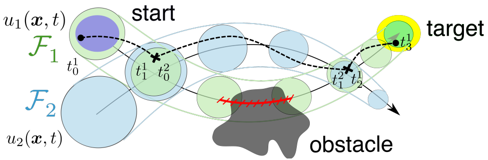


Corresponding timed automaton:



Our approach

More realistic idea: fixed set of funnels for control law + property



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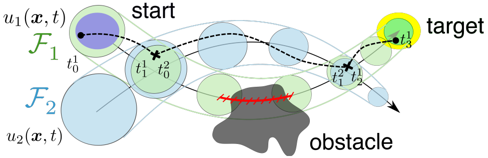

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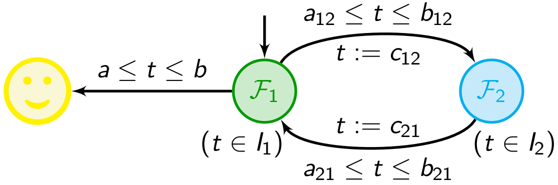
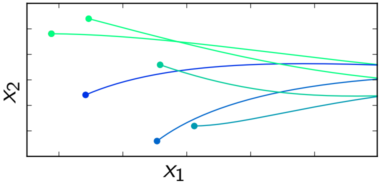


Control funnels

System with continuous dynamics $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, t)$



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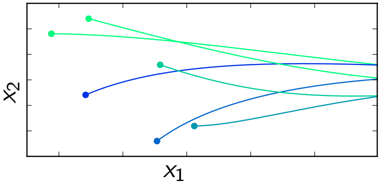

A (control) funnel is a trajectory $\mathcal{F}(t)$ of a set in the state space such that, for any trajectory $\boldsymbol{x}(t)$ of the dynamical system:

$$\forall t_0 \in \mathbb{R},\ \boldsymbol{x}(t_0) \in \mathcal{F}(t_0) \Rightarrow \forall t \geq t_0,\ x(t) \in \mathcal{F}(t)$$

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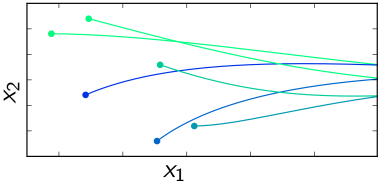

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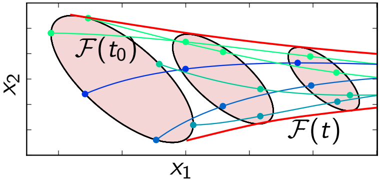
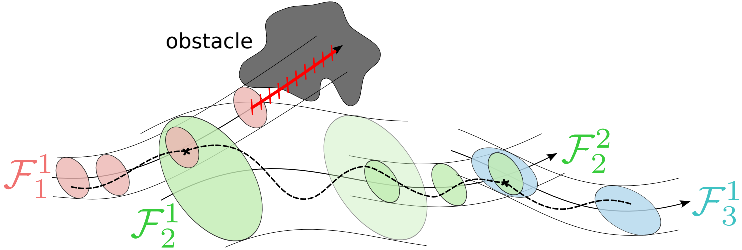


Example



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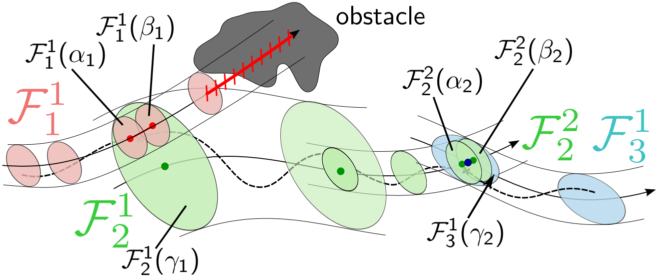
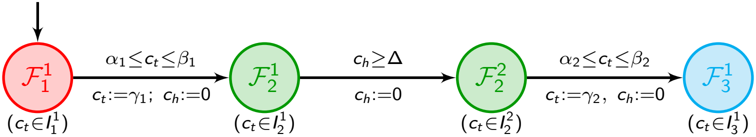
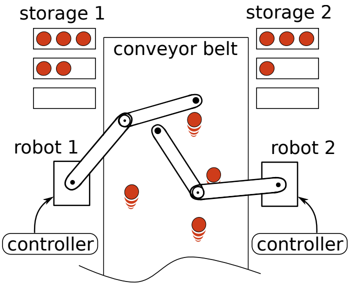


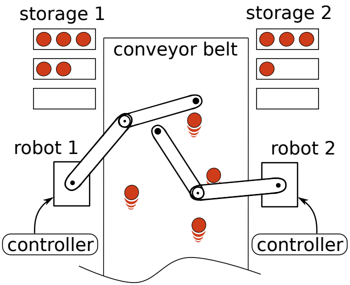
c_t: positional clock; c_h: local clock



Summary

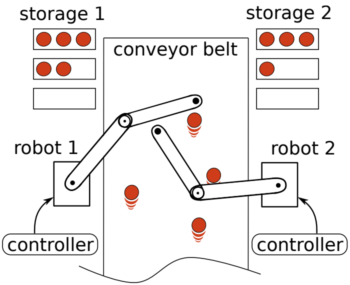

storage 1
storage 2
conveyor belt
robot 1
robot 2
controller
controller

Summary



$\rightsquigarrow$ (huge) timed automata/games (with weights), with few clocks

Summary



⇝ (huge) timed automata/games (with weights), with few clocks

← winning (optimal) strategy

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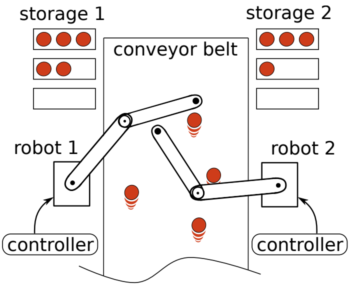
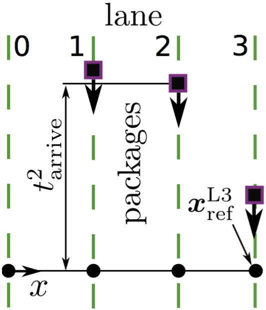


$\rightsquigarrow$ (huge) timed automata/games (with weights), with few clocks

safe (good) controller $\leftarrow$ winning (optimal) strategy

A pick-and-place example

1d point mass



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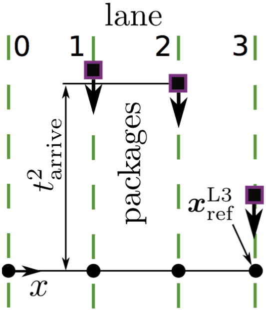
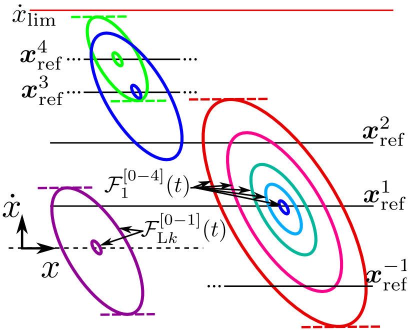
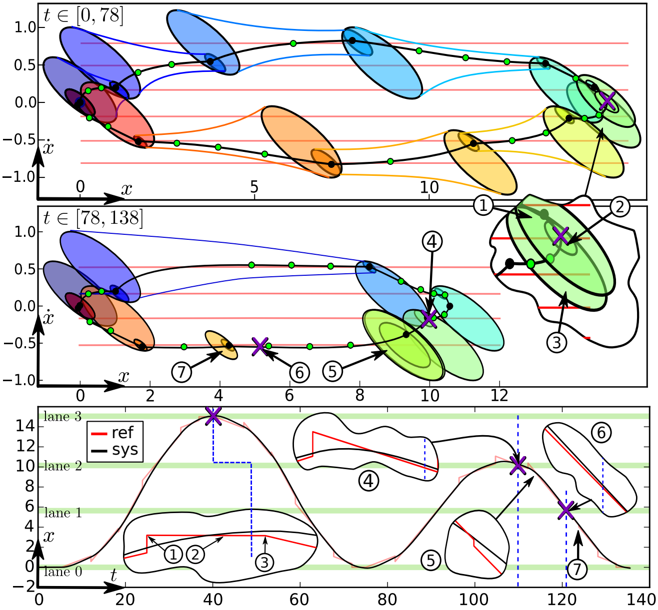


Funnel system





$t \in [0, 78]$
$t \in [78, 138]$
$\dot{x}$
x
t
lane 0
lane 1
lane 2
lane 3
ref
sys
1
2
3
4
5
6
7

Current challenges

For control people

- Handle more non-linear systems (automatically build control funnels)

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- Handle more non-linear systems (automatically build control funnels)

For us

- Does not scale up very well so far (huge timed automata models)
  - Build the model on-demand?
    But, can we give guarantees (optimality) when only part of the model has been built?
  - Develop specific algorithms for the special timed automata we construct?
- Implement efficient approx. algorithm for weighted timed games

Outline

Conclusion

Summary of the talk

- Overview of results concerning the optimal reachability problem in weighted timed automata and games
- Our new tool TiAMo

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Future work

- Various theoretical issues
  - Apply further the idea of approximation
  - Robustness issues

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- Overview of results concerning the optimal reachability problem in weighted timed automata and games
- Our new tool TiAMo

Future work

- Various theoretical issues
  - Apply further the idea of approximation
  - Robustness issues
- Continue working on TiAMo
  - Implementation of (weighted) timed games (good data structures, abstractions, etc.)
  - More applications with specific challenges (e.g. robotic problems)